Proton and Ion Linear Accelerators

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Proton and Ion Linear Accelerators

13. RF accelerating structures, Lecture 1

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Education in Beam Physics and Accelerator Technology

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Vyacheslav P. Yakovlev received MS degree in accelerator physics from Novosibirsk State University (NSU), Russia, in 1977, and PhD in accelerator physics from Budker Institute for Nuclear Physics (Budker INP), Novosibirsk, Russia, in 1988, where he worked as a Research scientist and since 1988 as a Senior Scientist. From 1994 to 1996 he was an Associate Professor at Novosibirsk State Technical University. Since 1996 he worked at Yale Beam Lab, Physics Department, Yale University, and Omega-P Inc as a Senior Scientist. Since 2007 he works at Fermilab as a Senior Scientist. Since 2011 to 2021 he was the Head of SRF Development Department at Application Science and Technology Division of Fermilab. Since 2021 to present he is the Head of Quantum Microwave System Department, Superconducting Quantum Materials and Systems Division of Fermilab. From 2017 to present he is an Adjunct Professor of Accelerator Science, Facility for Rare Isotope Beams, Michigan State University, Lansing, USA.

The scope of his professional interest includes physics and techniques of particle accelerators, namely: theory and simulations of the fields and beam dynamics in linear and circular accelerators; physics and technique of RF accelerator structures including room temperature cavities and structures, superconducting cavities and ferrite-tuned cavities; high power RF systems and RF sources for accelerators; tuning systems and cryo-module design, SRF for Quantum Computers. Over 400 publications.

RF accelerating structures

Outline:

- Introduction;
- Accelerating, focusing and bunching properties of RF field;
- RF Cavities for Accelerators

"Repetitio est mater studiorum"
(Repetition is the mother of learning)

Chapter 1.

Introduction.

Accelerators for scientific applications.

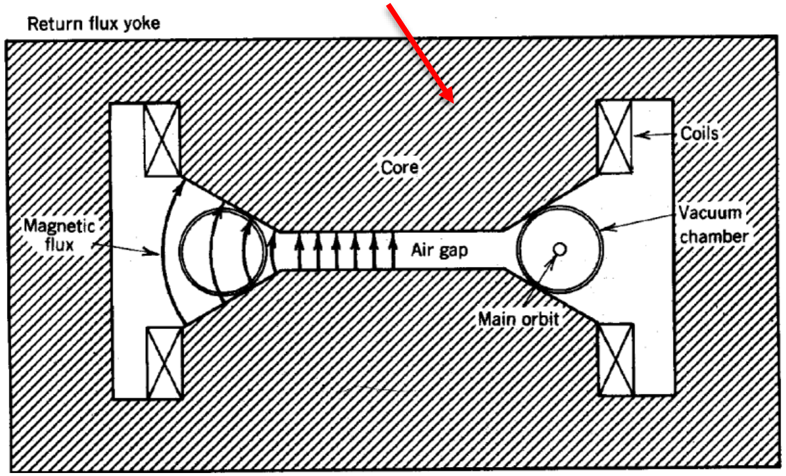
High – Energy Electron accelerators: High Energy Physics, Nuclear Physics, Free-Electron Lasers

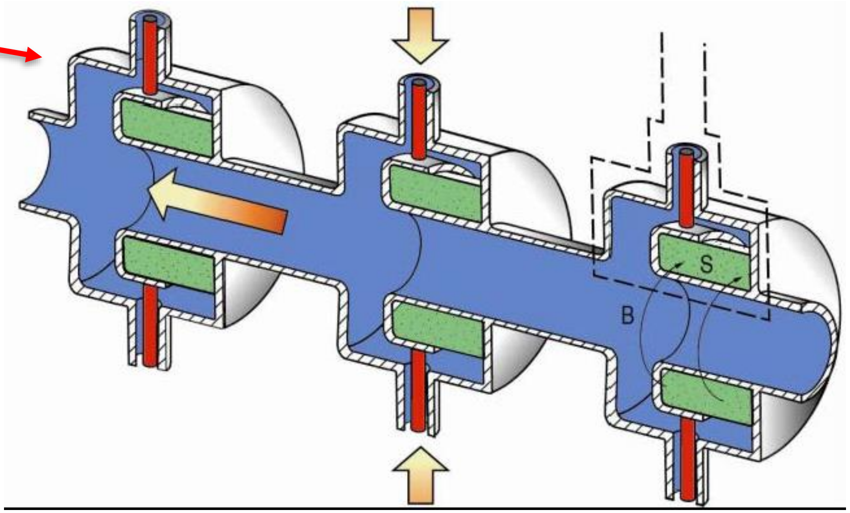
High – Energy Proton accelerators: High Energy Physics, Nuclear Physics, source of secondary particles (neutrons, pions, muons, neutrinos), material science, Accelerator-Driven Subcritical reactors (ADS).

Specifics of proton accelerators: protons are non- or weekly relativistic up to high energies: rest mass for protons is 0.938 GeV (compared to 0.511 MeV for electrons).

Types of the accelerators*

- **Electrostatic accelerators – acceleration in DC field**
  - Van de Graaff (moving belt to charge the high voltage electrode)
  - Cockcroft-Walton (diode-capacitor voltage multiplier).
- **Electrodynamic accelerators – acceleration in changing EM field.**
- Induction accelerators – acceleration in pulsed eddy electric field
  - *Linear induction accelerator*
  - *Cyclic induction accelerator*
    - Betatron





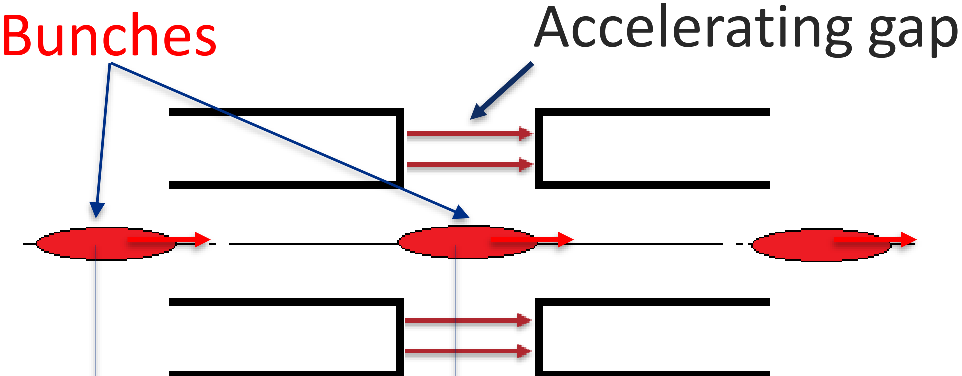
$$e\oint \vec{E} \bullet d\vec{l} = -e\partial/\partial t \int_s \vec{B} \bullet d\vec{S}$$

*P. Ostroumov, "Introduction to accelerators," **PHY862 Accelerator Systems, 2020**

Types of linear accelerators

❖ RF accelerators – acceleration in RF field

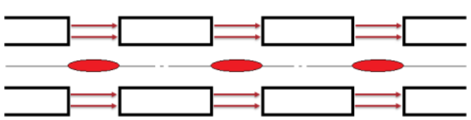
- Bunched beam (no particles when the field is decelerating);
- Accelerating RF field is excited in an accelerating gap;



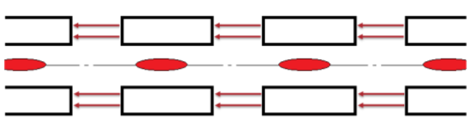
Particle energy gain $\Delta U \sim E_0$

RF field: $E = E_0 \cos(\omega t)$

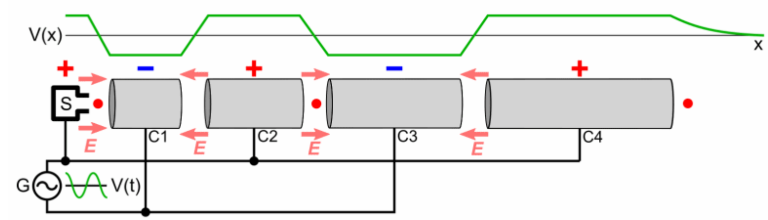
$L = nvT = 2\pi nv/\omega$, v is the bunch velocity, T is RF period and ω is RF cyclic frequency; n is integer, n=1,2…



$\omega t = 2\pi m$, m=0,1,2…



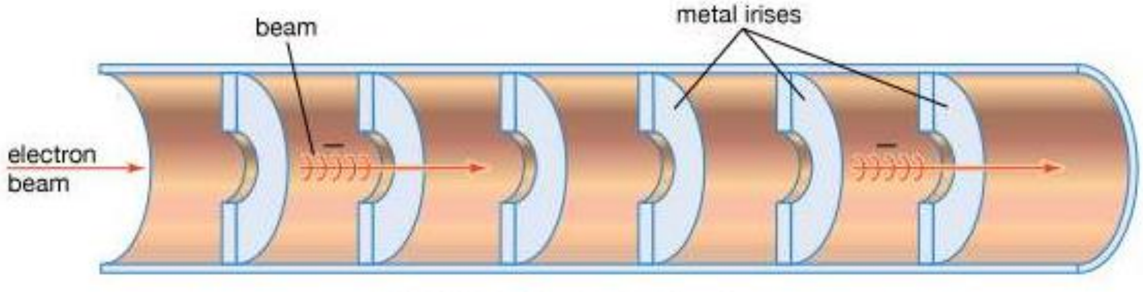
$\omega t = (2m+1)\pi$, m=0,1,2…



Animation from Wikipedia
(https://en.wikipedia.org/wiki/Particle_accelerator)

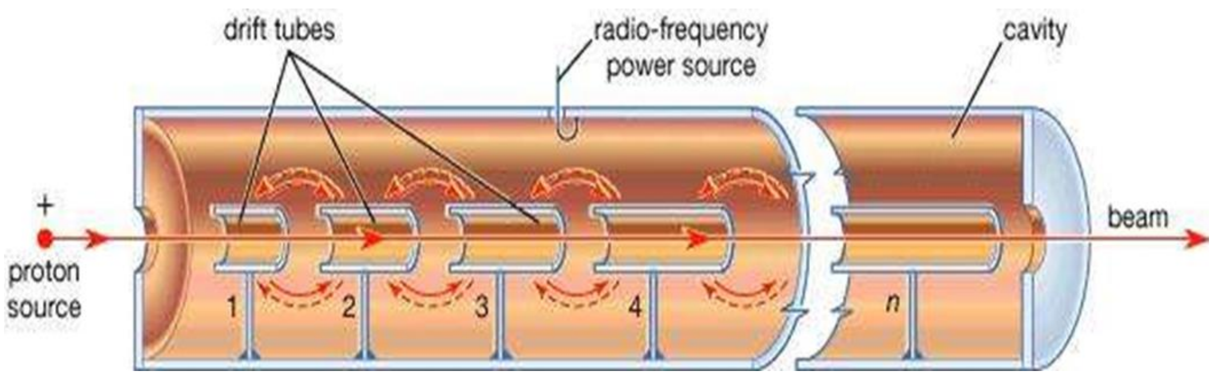
RF linear accelerators

- Travelling wave accelerators.



The wave propagates left to right.

- Standing wave accelerators.

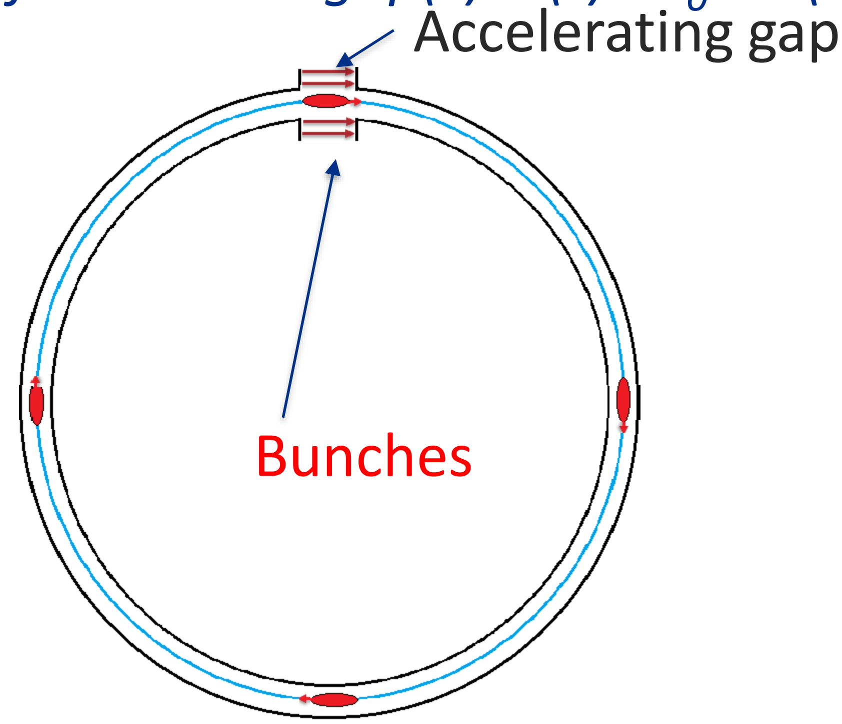


- Room Temperature linear accelerators
- Superconducting linear accelerators

Types of the accelerators*

- An accelerated particle passes the same gap many times (cyclic accelerator).

RF fields in the gap(s): $E(t) = E_0 \cos(\omega t)$



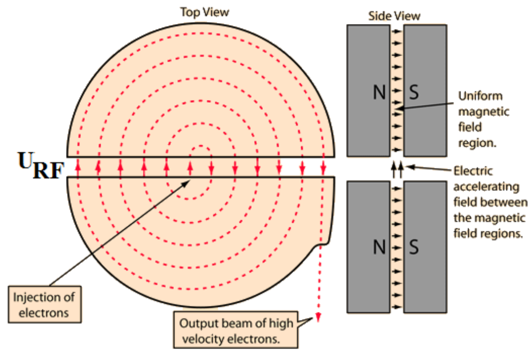
$\omega = \Omega N q;$

ω is RF frequency,
Ω is the bunch revolution frequency,
N is number of bunches
q=1,2,3…

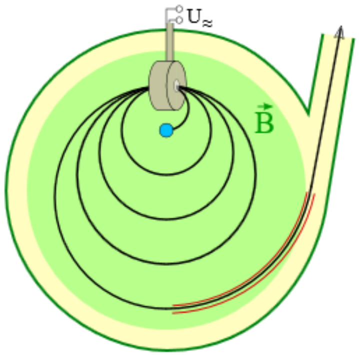
$\omega t = 2\pi m,\ m = 0,1,2\ldots$

*P. Ostroumov, "Introduction to accelerators," **PHY862 Accelerator Systems, 2019**

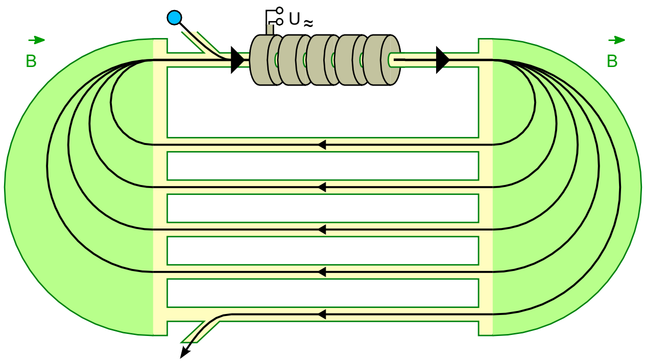
Cyclic accelerators



Cyclotron

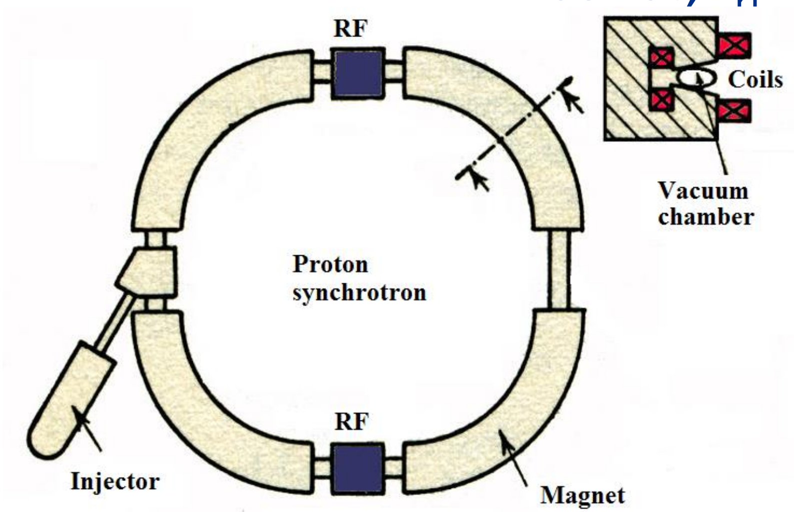


Microtron



Racetrack

B= const, f_{rf} = const



Proton synchrotron:

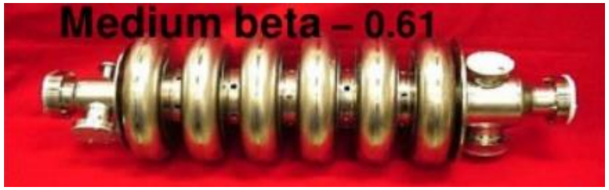
B=B(t), f_{rf}=f_{rf}(t)

*P. Ostroumov, "Introduction to accelerators," **PHY862 Accelerator Systems, 2019**

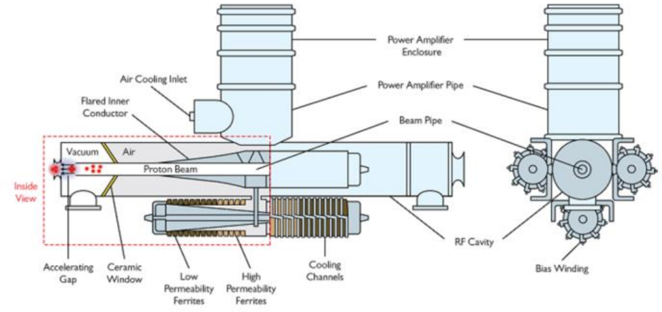
RF cavities for accelerators

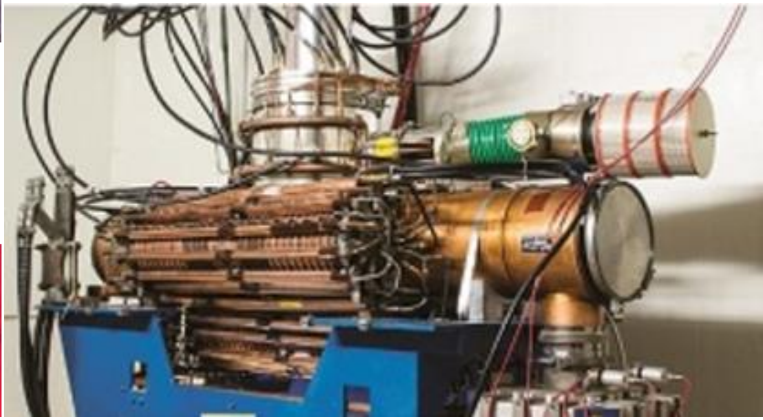
To achieve high accelerating field in the gap resonant RF cavities are used. In all modern RF accelerators, the beam acceleration takes place in a resonance wave (standing or travelling) electromagnetic field excited in an RF cavities.

CW 50.6 MHz cavity of PSI cyclotron. V =1.2 MV



Superconducting 805 MHz multi-cell cavities of SNS linac. V=10-15 MV, DF = 6%.





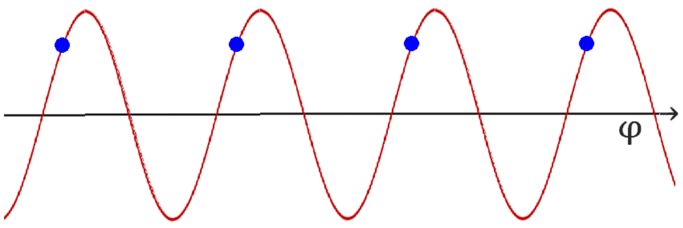
Tunable cavity for FNAL Booster Synchrotron
F=37.8-52.8 MHz. V=60 kV, DF =50%

Acceleration principles:

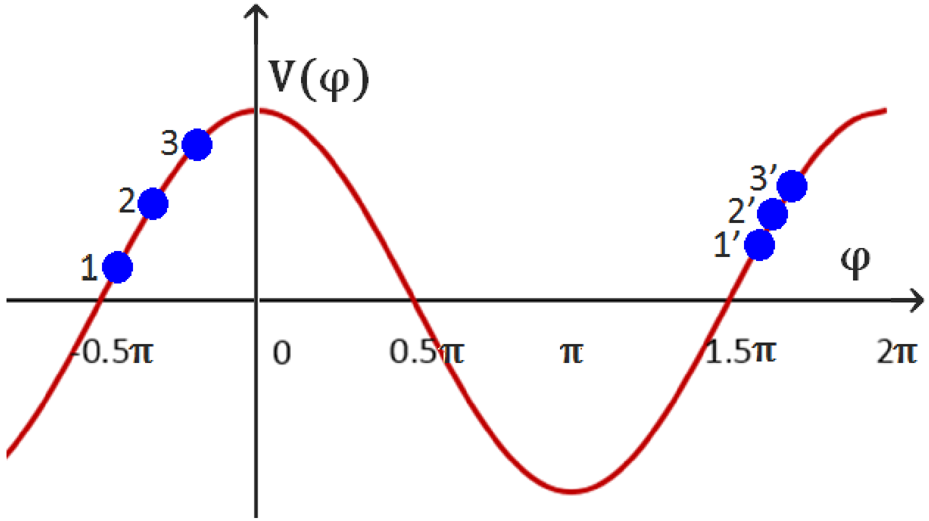
If the charged particle reaches the center of the accelerating gap in arbitrary phase φ, its energy gain is

$$V(\varphi) = V\cos(\varphi)$$

Acceleration: $-\pi/2 < \varphi < \pi/2$



- ***Synchronism:*** the bunches should reach the center of the acceleration gaps in the same accelerating phase.
- ***Autophasing:*** longitudinal dynamics should be stable (no bunch lengthening). For linear accelerator $-\pi/2 < \varphi_s < 0$ (φ_s is the phase of the bunch center)



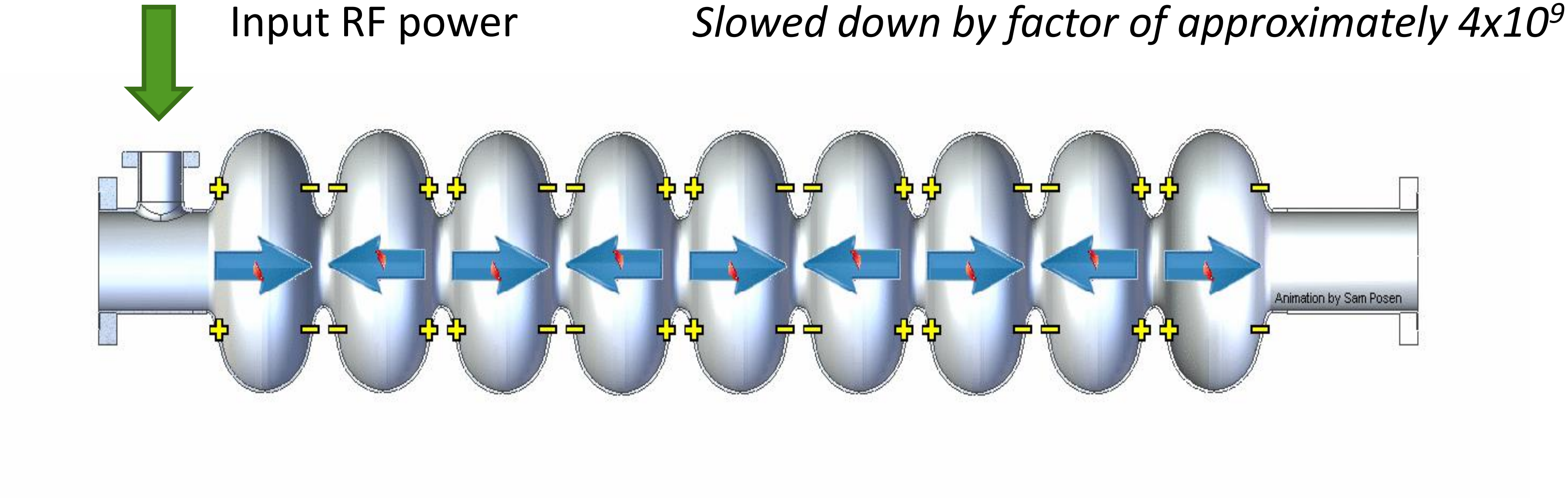
(linear accelerator)

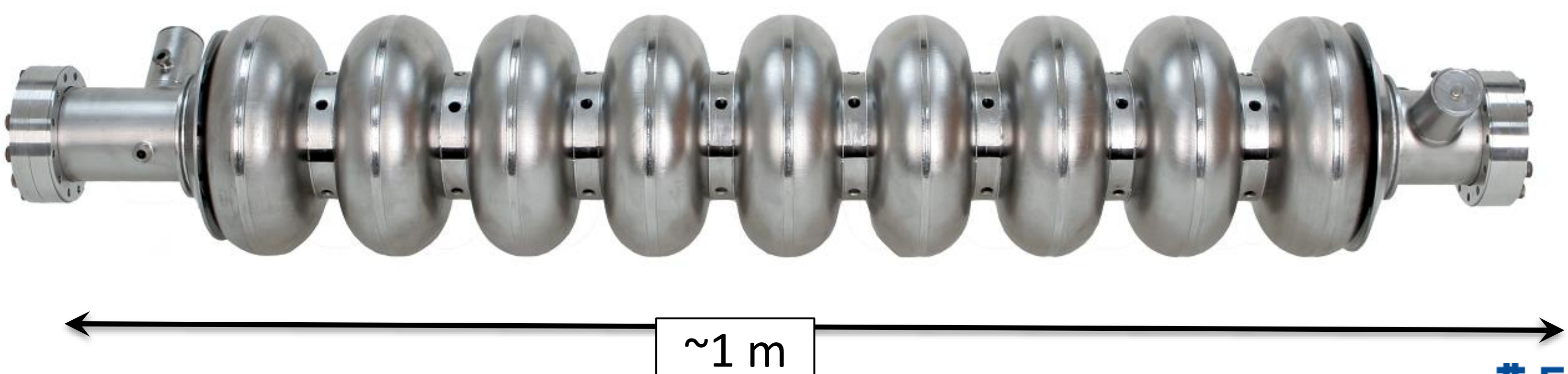
Illustration of synchronism :

1.3 GHz ILC cavity (animation by Sam Posen, FNAL)

Input RF power

Slowed down by factor of approximately $4x10^9$





Chapter 2.

Accelerating and focusing properties of RF field.

a. Acceleration of charged particles in electromagnetic field;
b. Focusing properties of RF field;
c. Bunching properties of RF field;
d. Summary.

Acceleration and focusing of charged particles in electromagnetic field

Electromagnetic fields in RF cavities are described by Maxwell equations:

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \mathrm{rot}\,\vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}. \quad \frac{\partial \rho}{\partial t} + \mathrm{div}\,\vec{J} = 0,$$

$$\mathrm{div}\,\vec{B} = 0, \quad \mathrm{div}\,\vec{D} = \rho.$$

$$\vec{D} = \varepsilon \vec{E}, \quad \vec{B} = \mu \vec{H}, \quad \vec{J} = \sigma \vec{E}.$$

$$\nabla \equiv \sum_{i=1}^{3} \vec{e}_i \frac{\partial}{\partial x_i}$$

$$grad\ f \equiv \nabla f$$

$$div\ \vec{A} \equiv \nabla \cdot \vec{A}$$

$$rot\ \vec{A} \equiv curl\ \vec{A} \equiv \nabla \times \vec{A}$$

Harmonic oscillations:

$$\vec{E} = \vec{E}(r) \cdot \mathrm{e}^{\,i\omega t}, \quad \mathrm{rot}\,\vec{E} = -i\omega\mu\vec{H}, \quad \mathrm{rot}\,\vec{H} = i\omega\varepsilon\vec{E} + \vec{J}.$$

For vacuum:

$$\mu_0 = 4\pi \cdot 10^{-7}\,\frac{H}{m}; \quad \varepsilon_0 = \frac{10^{-9}}{36\pi} \approx 0.884 \cdot 10^{-11}\,\frac{F}{m}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,\mathrm{Ohm}; \quad \frac{1}{\sqrt{\varepsilon_0\,\mu_0}} = c$$

Acceleration and focusing of charged particles in electromagnetic field

From Maxwell equations:

$$-\mathrm{rot}\,\mathrm{rot}\,\vec{E} = -\omega^2\varepsilon\mu\vec{E} + i\,\omega\mu\vec{J}.$$

For $\vec{J} = 0$

$$\mathrm{rot}\,\mathrm{rot}\,\vec{E} = \omega^2\varepsilon\mu\vec{E}$$ or

$$\Delta\vec{E} + k^2\vec{E} = 0,\ k^2 = \omega^2\varepsilon\mu\ .$$

Same for magnetic field:

$$\Delta\vec{H} + k^2\vec{H} = 0.$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$ Cartesian

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}.$$ Cylindrical

Useful theorems are in Appendix 1

Acceleration and focusing of charged particles in electromagnetic field

❑ Boundary conditions on a conductive wall:

$$\vec{E}_t = Z_s(k)[\vec{H}_t \times \vec{n}],$$

where $Z_s(k)$ is a surface impedance, $\vec{n}$ is directed to the metal.

- Wall power loss:

$$P = \frac{1}{2} Re \int (\vec{E} \times \vec{H}) \vec{n} dS = \frac{1}{2} \int R_s |H|_t^2 dS,$$

R_s is the surface resistance,

$$R_s = Re(Z_s(k))$$

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Acceleration and focusing of charged particles in electromagnetic field

1. Normal-conducting metal, classic skin effect.

- Surface impedance

$$Z_s(k) = \sqrt{\frac{kZ_0}{2\sigma}}(1-i)$$

where σ is the wall material conductivity. For copper at room temperature (20°C) σ = 59 MS/m.

- Surface resistivity:

$$R_s = Re[Z_s(k)] = \sqrt{\frac{kZ_0}{2\sigma}} = \sqrt{\frac{\omega Z_0}{2c\sigma}}.$$

$$rot\vec{E} = -i\omega\vec{B}, rot\vec{H} = i\omega\vec{D} + \vec{J} \approx \sigma\vec{E}$$

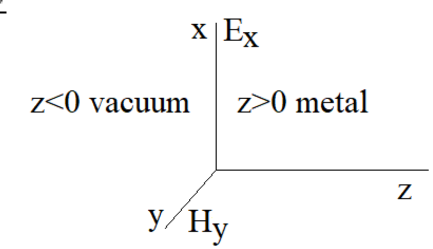
$$\downarrow$$

$$rotrot\vec{H} = -i\omega\mu_0\sigma\vec{H} \rightarrow \frac{d^2H_y}{dz^2} = -i\omega\mu_0\sigma H_y = -ikZ_0\sigma H_y$$

$$\downarrow$$

$$H_y(z) = H_s e^{-(1-i)z/\delta},\ \delta = \sqrt{\frac{2}{kZ_0\sigma}};$$

$$\frac{E_x}{H_y} = Z_s(k) = (1-i)\frac{1}{\delta\sigma} = (1-i)\sqrt{\frac{kZ_0}{2\sigma}}$$



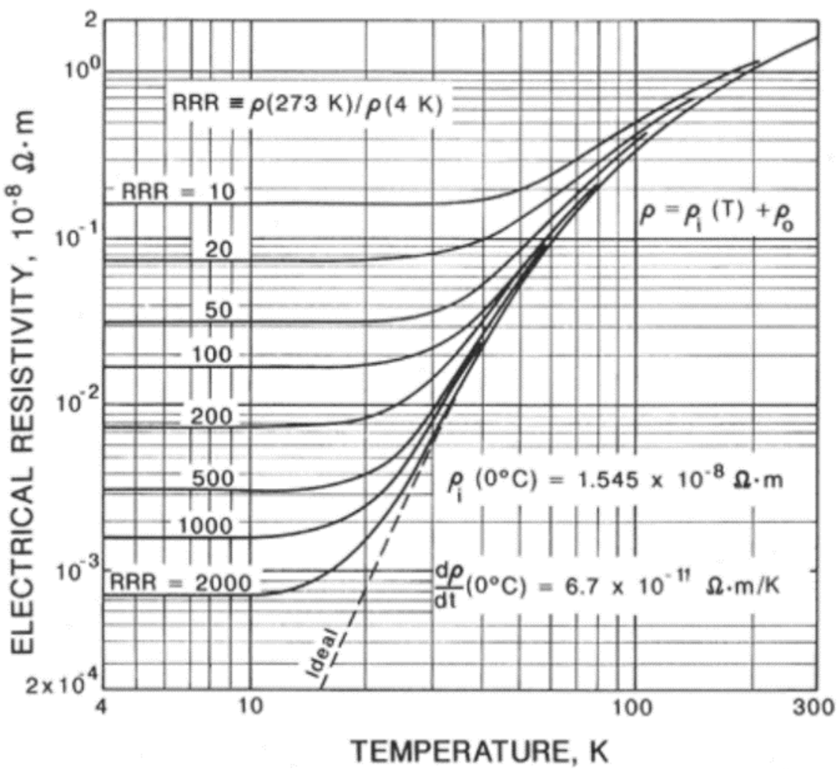
- The RF field *H(z)* and *E(z)* decays into the metal exponentially with the distance from the surface *z*:

$$\frac{H(z)}{H_s} = \frac{E(z)}{E_s} = e^{-(1-i)z/\delta},\ \delta = \sqrt{\frac{2}{kZ_0\sigma}}$$ - classical skin depth.

Acceleration and focusing of charged particles in electromagnetic field

- For pure metals, the conductivity decreases with the temperature.

Copper resistivity $\rho = \sigma^{-1}$ versus temperature for different sample purity:



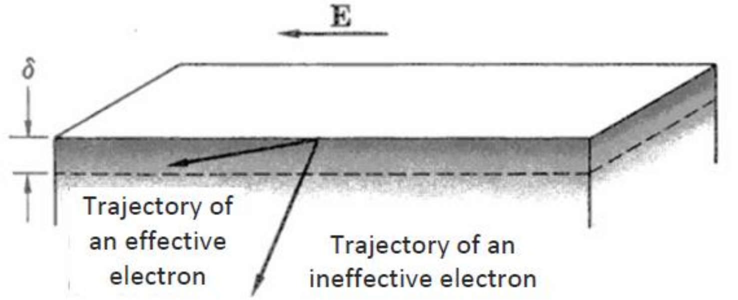
A commonly used measure of purity is the residual resistivity ratio (RRR), defined as the ratio of the resistivity at 273 K or 0°C over the resistivity at 4K:

$$RRR = \frac{\rho(273\ K)}{\rho(4K)}$$

Acceleration and focusing of charged particles in electromagnetic field

2. Normal-conducting metal, anomalous skin effect*:

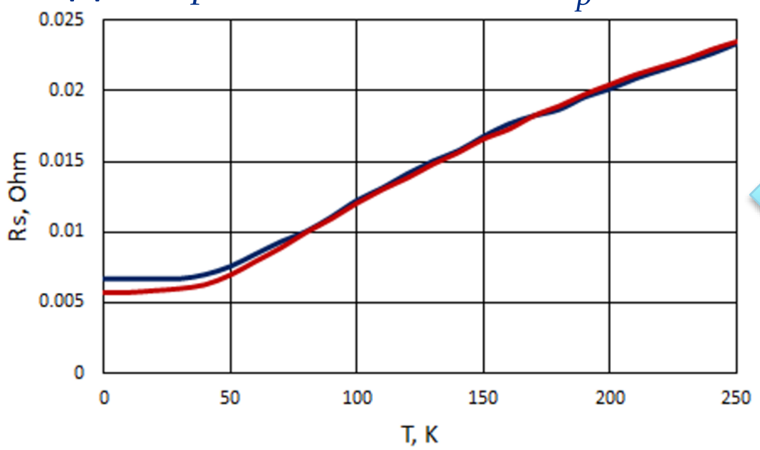
- At low temperature of metal skin depth δ may be smaller than the mean-free path l of conducting electrons l. **It is anomalous skin effect.**



- For extreme anomalous skin effect the surface resistance Z_s may be estimated as

$$Z_s = Z_0 \left(\frac{\sqrt{3}}{16\pi}\frac{c v_F k^2}{\omega_p^2}\right)^{1/3}\left(1-\sqrt{3}i\right), \qquad R_s = Re(Z_s) = \left(\frac{\sqrt{3}}{16\pi}\frac{c v_F k^2}{\omega_p^2}\right)^{1/3},$$

where v_F is the Fermi velocity and ω_p is the plasma frequency of conducting electrons. For copper v_F = 1.58e6 m/sec, ω_p = 1.64e16 rad/sec and l=39 nm; $\frac{3}{2}\left(\frac{l}{\delta}\right)^2 \gg 1,\ \omega \ll \omega_p$.



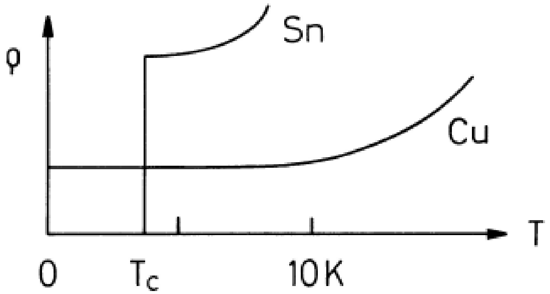
Calculated pure copper surface resistance (red) versus measured (blue) for the frequency of 11424 MHz. Mean-free path for pure copper is 39 nm.

*CHAMBERS, R. Anomalous Skin Effect in Metals. *Nature* **165,** 239–240 (1950).

Acceleration and focusing of charged particles in electromagnetic field

2. Superconducting wall:

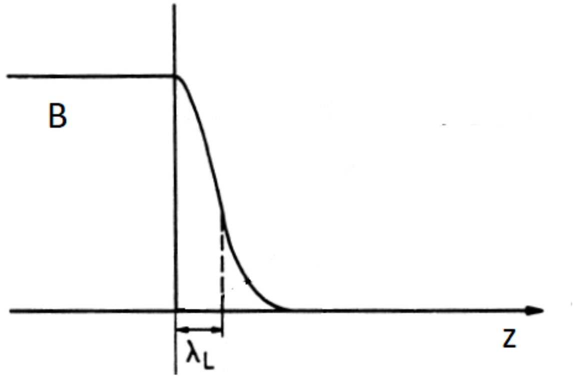
- Superconductivity - the infinitely high conductivity (or zero resistivity) below a 'critical temperature' T_c.



T_c(K):

Al	Hg	Sn	Pb	Nb	Ti	NbTi	Nb_3Sn
1.14	4.15	3.72	7.9	9.2	0.4	9.4	18

- Penetration depth λ_L: $B(z) \sim exp(-z/\lambda_L)$



material	In	Pb	Sn	Nb
λ_L [nm]	24	32	≈ 30	32

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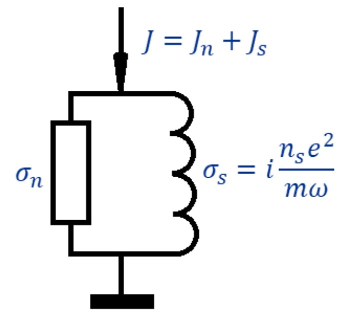
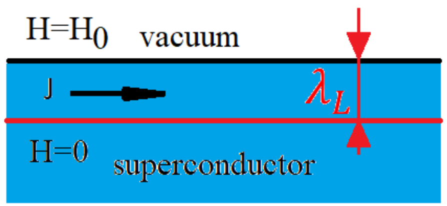
Acceleration and focusing of charged particles in electromagnetic field

- Two-fluid model: current in a superconductor is carried by
  - the superfluid component (Cooper pairs) - J_s;
  - the normal component (unpaired electrons) - J_n.

○ At DC – no resistance.

○ At AC – resistance caused by electron inertia.





$$Z_s = \frac{1}{\lambda_L(\sigma_n + \sigma_s)}$$

For normal component:

$J_n = \sigma_n E_0 \exp(-i\omega t)$,

For superfluid component:

$$m\dot{v} = -eE_0 \exp(-i\omega t) \rightarrow J_s = -en_s v = i\frac{n_s e^2}{m\omega} E_0 \exp(-i\omega t) =$$

$$= \sigma_s E_0 \exp(-i\omega t) \rightarrow \sigma_s = i\frac{n_s e^2}{m\omega}.$$

$R_s = Re\left(\frac{1}{\lambda_L(\sigma_n+\sigma_s)}\right) \approx \frac{1}{\lambda_L} \cdot \frac{\sigma_n}{|\sigma_s|^2}$, or $R_s \propto \omega^2 \exp\left(-\frac{\Delta}{k_B T}\right)$, because

$\sigma_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$ and $|\sigma_s|^{-2} \propto \omega^2$. Here $\Delta \sim T_c$ is the energy gap and k_B is the Boltzmann constant.

Phenomenological law for Nb:

$$R_{s,BCS} \approx 1.643 \times 10^{-5} \frac{T_c}{T} (f(GHz))^2 e^{-\frac{1.92 T_c}{T}} \ (\Omega).$$

$$R_s = R_{s,BCS} + R_{residual}$$

Acceleration and focusing of charged particles in electromagnetic field

Examples:

1. Surface resistance of a copper wall at room temperature for 1.3 GHz

$R_s = \sqrt{\frac{\omega Z_0}{2c\sigma}}$= 9.3 mOhm;

σ=59 MS/m; ω=2π·1.3e9 Hz, Z_0=120π Ohm, c=3e8 m/sec.

2. Surface resistance of a pure copper (RRR=2500) wall at 2 K for 1.3 GHz

$R_s = \left(\frac{\sqrt{3}}{16\pi}\frac{c v_F k^2}{\omega_p^2}\right)^{1/3}$ = 1.3 mOhm.

v_F = 1.58e6 m/sec, ω_p = 1.64e16 rad/sec, k= ω/c= 2π·1.3e6/c.

Mean-free path is 39 nm compared to classical skin depth of 36 nm (!).

Classical skin formula gives 0.19 mOhm!

3. BCS resistance of the Nb at 2 K.

$R_{s,BCS} \approx 1.643 \times 10^{-5}\frac{T_c}{T}(f(GHz))^2 e^{-\frac{1.92 T_c}{T}}$ = 19 nOhm

f=1.3 GHz, Tc =9.2 K, T=2K.

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Acceleration and focusing of charged particles in electromagnetic field

Dynamics of a charged particle accelerated in a RF field is described by Lorenz equation,

$$\frac{d\vec{p}}{dt} = \vec{F} = e[\vec{E}_0(\vec{r},\mathrm{t}) + \vec{v} \times \vec{B}_0(\vec{r}, t)]\,,$$

where $\vec{v}$ is the particle velocity, $\vec{E}_0(\vec{r},\mathrm{t})$ and $\vec{B}_0(\vec{r}, t)$ are RF electric and magnetic field oscillating at the cavity resonance frequency ω:

$$\vec{E}_0(\vec{r},\mathrm{t}) = \vec{E}(\vec{r})\, e^{i\omega \mathrm{t}}$$

$$\vec{B}_0(\vec{r},\mathrm{t}) = \vec{B}(\vec{r})\, e^{i\omega \mathrm{t}}$$

Acceleration and focusing of charged particles in electromagnetic field

RF electric field has a longitudinal component next to the beam axis. In cylindrical coordinate it may be expanded over azimuthal harmonics, i.e.,

$$E_z(r,\varphi,z) = \sum_{m=-\infty}^{\infty} e^{im\varphi} E_{m,z}(r,z)$$

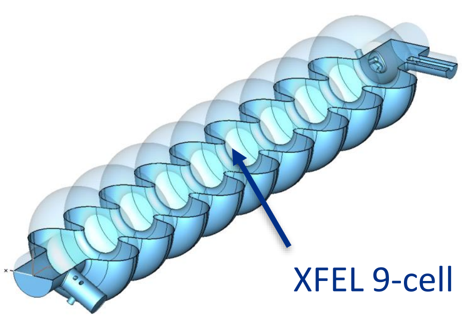
To understand general properties of the acceleration field, the amplitudes may be expanded into Fourier integral for $r < a$, a is the beam aperture :

$$E_{m,z}(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_{m,z}(k_z,r)\, e^{ik_z z} dk_z \qquad (1)$$

or over the travelling waves existing from z=-∞ to z=∞.

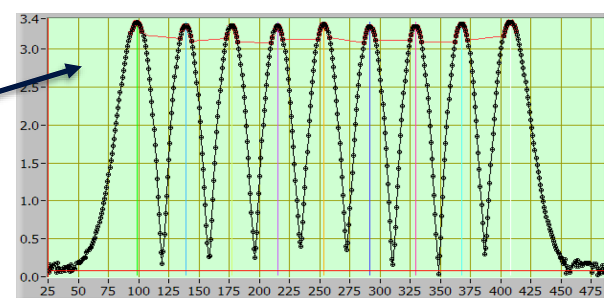
The RF field $\vec{E}_0(\vec{r},\mathrm{t})$ satisfies the wave equation:

$$\Delta\vec{E}_0(\vec{r},\mathrm{t}) + \frac{\partial^2 \vec{E}_0(\vec{r},\mathrm{t})}{\partial t^2} = 0 \qquad (2)$$



XFEL 9-cell 1.3 GHz SW cavity

$|E_{0,z}(0,z)|$



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Acceleration of charged particles in electromagnetic field

Substituting expansion (1) to the wave equation (1), we can find, that $E_{m,z}(k_z, r)$ satisfies Bessel equation, and therefore is proportional to the Bessel function $J_m(x)$,

$$E_{m,z}(k_z, r) = E_{m,z}(k_z) J_m(k_\perp r)$$

where $k_\perp$ is transverse wavenumber, which it turn satisfies dispersion equation:

$$k_\perp^2 + k_z^2 = \frac{\omega^2}{c^2} \equiv k^2,$$

here c is speed of light.

If the particle velocity $v = \beta c$ and particle transverse coordinates do not change significantly in the cavity, the energy ΔW particle gains in the cavity is equal to

$$\Delta W(r, \varphi) = eRe\left[\int_{-\infty}^{\infty} dz E_z(r, \varphi, z)\, e^{i\omega t}\Big|_{t=\frac{z}{v}}\ \right]$$

Acceleration of charged particles in electromagnetic field

Performing integration over z one has:

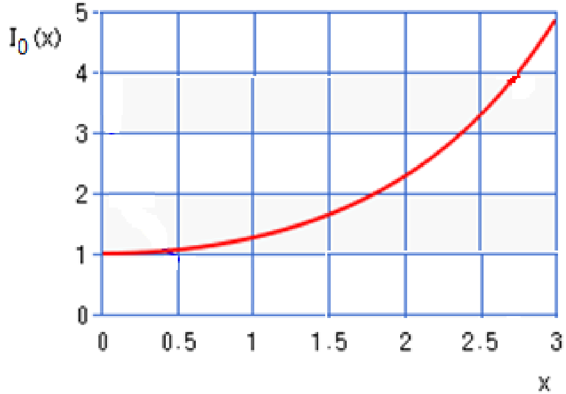
$$\Delta W(r,\varphi) = eRe[\sum_{m=-\infty}^{\infty} E_{m,z}\left(\frac{k}{\beta}\right) I_m\left({}^{kr}/_{\beta\gamma}\right) e^{im\varphi}\ ],$$

where $I_m(x)$ is modified Bessel function and γ is the particle relativistic factor (note that $k_\perp = ik/\beta\gamma$); i.e., the particle gains the energy interacting with synchronous cylindrical wave having the phase velocity equal to the particle velocity (synchronism: $v_{particle}=\beta c = v_{phase}=\omega/k_z \rightarrow k_z=k/\beta$ and $k_\perp = (k^2-k_z^2)^{1/2}= ik/\beta\gamma$.

If the cavity and RF field of the operating mode have perfect azimuthal symmetry, one has:

$$\Delta W(r) = eRe\,[E_{0,z}\left(\frac{k}{\beta}\right) I_0\left({}^{kr}/_{\beta\gamma}\right)] = e|E_{0,z}\left(\frac{k}{\beta}\right)|I_0\left({}^{kr}/_{\beta\gamma}\right)cos\phi$$

where ϕ is the RF phase.

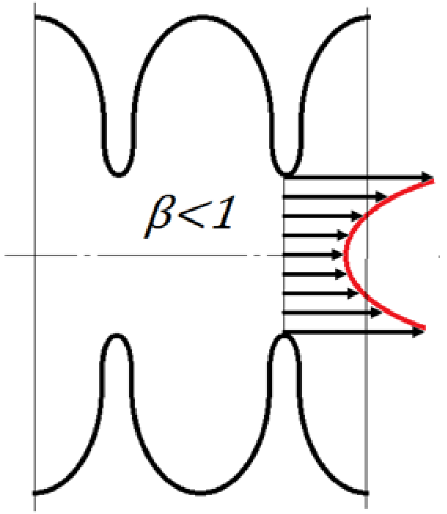


Acceleration of charged particles in electromagnetic field

- For a very slow particle, i.e., when $\beta << 1$, if $kr/\beta\gamma >> 1$ one has

$$I_0\left({}^{kr}/_{\beta\gamma}\right) \sim \frac{1}{\sqrt{kr}} e^{{}^{kr}/_{\beta\gamma}}.$$

It means that for low-beta particle the energy gain increases with the radius r.



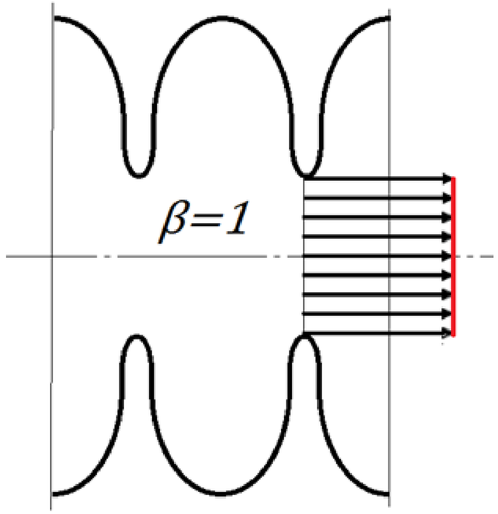
- For the ultra-relativistic particle $k_{\perp} \rightarrow 0$ and one has

$$\mathrm{V}(r,\varphi) = eRe[\sum_{m=-\infty}^{\infty} E_{m,z}(k)\, r^m e^{im\varphi}],$$

For the RF field having perfect azimuthal symmetry

$$\mathrm{V}(r) = e|E_{0,z}(k)|cos\phi$$

and the particle energy gain does not depend on the transverse coordinate.

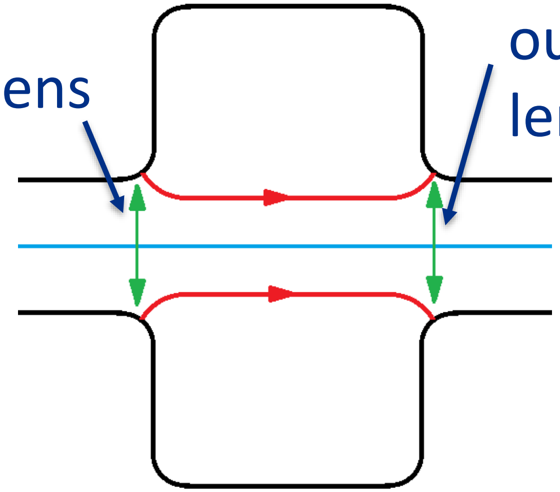


Focusing properties of RF field

In addition to acceleration, the RF field provides deflection of the beam. Let's consider the particle transverse momentum change causes by the cavity RF field. The particle moves on the trajectory $z=vt$ parallel to the axis, but has off-set $\vec{r}_\perp$. According to **Panofsky – Wenzel theorem (Appendix 2)**, change of transverse momentum caused by RF field is related to change of the longitudinal momentum:

$$\Delta\vec{p}_\perp = \frac{iv}{\omega}\vec{\nabla}_\perp(\Delta p_\parallel).$$

The differential operator $\vec{\nabla}_\perp$ acts on the transverse coordinates $\vec{r}_\perp$ only; longitudinal and transverse momentum changes are (Appendix 1):



$$\Delta p_\parallel = e\int_{-\infty}^{\infty} E_z(\vec{r})e^{i\omega t}dt\,|_{t=z/v} = \frac{e}{v}\int_{-\infty}^{\infty} E_z(\vec{r})e^{i\omega z/v}dz;$$

$$\Delta\vec{p}_\perp = e\int_{-\infty}^{\infty}\left[\vec{E}_\perp(\vec{r}) + \left(\vec{v}\times\frac{i}{\omega}rot\vec{E}(\vec{r})\right)_\perp\right]e^{i\omega t}dt\,|_{t=z/v} = e\int_{-\infty}^{\infty}\left[\vec{E}_\perp(\vec{r}) + \frac{iv}{\omega}\vec{\nabla}_\perp E_z(\vec{r}) - \frac{iv}{\omega}\frac{\partial\vec{E}_\perp(\vec{r})}{\partial z}\right]e^{i\omega t}dt\,|_{t=z/v}$$

No acceleration → no deflection!

Focusing properties of RF field

Therefore,

$$Re\Delta p_{\perp} = -\frac{e}{\omega}\sum_{m=-\infty}^{\infty} |E_{m,z}\left(\frac{k}{\beta}\right)| \cdot \vec{\nabla}_{\perp}[I_m\left({}^{kr}/_{\beta\gamma}\right) \cdot \cos(m(\varphi - \varphi_m))] \cdot \sin\phi$$

where φ_m is polarization of the azimuthal harmonics. The maximum of transverse momentum change is shifted in RF phase versus the maximum the energy gain by -90°: for the particle accelerated on crest of the RF field, transverse momentum change is zero. In order to get longitudinal stability in low-energy accelerator one needs to accelerate the particle at $\phi < 0$. One can see that for the field having perfect azimuthal symmetry

$$Re\Delta p_{\perp} = -\frac{e}{\omega}|E_{0,z}\left(\frac{k}{\beta}\right)| \cdot \vec{\nabla}_{\perp}[I_0\left({}^{kr}/_{\beta\gamma}\right)] \cdot sin\phi = -\frac{1}{\omega}\boldsymbol{V}(0) \cdot I_1\left({}^{kr}/_{\beta\gamma}\right) \cdot sin\phi$$

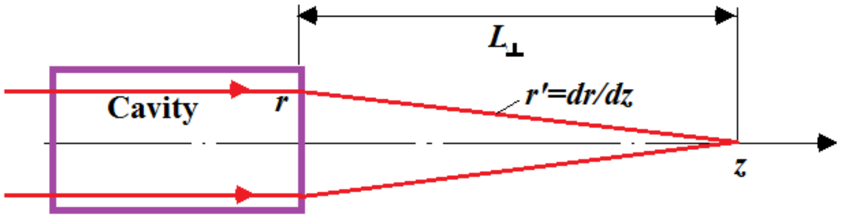
Near the axis, where ${}^{kr}/_{\beta\gamma} \ll 1$ one has for the trajectory angle r'in the end of acceleration $r' = \frac{\Delta p_{\perp}(r)}{p_{||}} \approx -\frac{kr}{2\beta^3\gamma^3}\frac{V_{max}(0)}{m_o c^2/\boldsymbol{e}} \cdot sin\phi$,

where m_o is the particle rest mass.

Focusing properties of RF field

❖ The focusing distance $L_\perp$ is

$$L_\perp = -\frac{r}{r\prime} = \frac{2\beta^3\gamma^3}{\omega/c}\frac{m_o c^2/e}{V_{max}(0)\cdot sin\phi}$$



- Focusing distance $L_\perp$ is inversed proportional to the RF frequency and proportional to β^3 . Because of this, at low energies the cavity provides strong defocusing ($\phi < 0$!), and this defocusing should be compensated by external magnetic focusing system.
- To mitigate this defocusing, one should use lower RF frequency ω in low energy parts of the linac ($L_\perp \sim 1/\omega$).

❖ For an ultra-relativistic particle in this case one has:

$$Re\Delta p_\perp = -\frac{e}{\omega}\sum_{m=-\infty}^{\infty}|E_{m,z}(k)| \cdot mr^{m-1} \cdot sin\phi$$

and in the case of perfect azimuthal symmetry of the field $\Delta p_\perp = 0$. However, the RF field provides transfer momentum change for ultra-relativistic particle, i.e., focusing. The reason is that the particle transverse coordinate and energy change during acceleration because of the initial trajectory angle and influence of the RF field.

In this case the transverse momentum change is proportional to the RF amplitude squared.

Focusing properties of RF field

The transport matrix which determines relationship between the input and output transverse coordinates and angles (x and x' respectively) of the relativistic particle is calculated, for example, in [*]. For the RF cavity operating at π-mode (slide) for the particle accelerated on crest, the transport matrix is the following:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_f = \begin{bmatrix} \cos(\alpha) - \sqrt{2}\sin(\alpha) & \sqrt{8}\,\frac{\gamma_i}{\gamma'}\sin(\alpha) \\ -\frac{3\gamma'}{\sqrt{8}\,\gamma_f}\sin(\alpha) & \frac{\gamma_i}{\gamma_f}[\cos(\alpha) + \sqrt{2}\sin(\alpha)] \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{in},$$

where $(x,x')_i$ initial coordinate and angle, $(x,x')_f$ are final parameters, γ_i and γ_f are initial and final relativistic factors, γ' is the acceleration gradient over the rest mass in electron-Volts ($\gamma' = \Delta W_{max}/L_c m_o c^2$ (L_c is the cavity length) and $\alpha = \frac{1}{\sqrt{8}} \ln\frac{\gamma_f}{\gamma_i}$.

- Note, that the angle x_f' at the cavity output for $x_i' = 0$ is proportional to the gain over the particle energy squared:

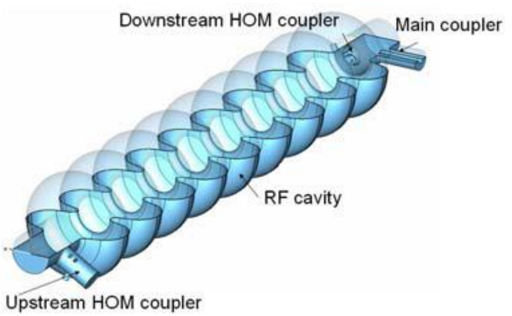
$$x'_f = \frac{\Delta p_\perp}{p_{||}} \approx \frac{3}{8}\left(\frac{V_{max}}{\gamma m_0 c^2}\right)^2 \frac{x_i}{L_c}$$

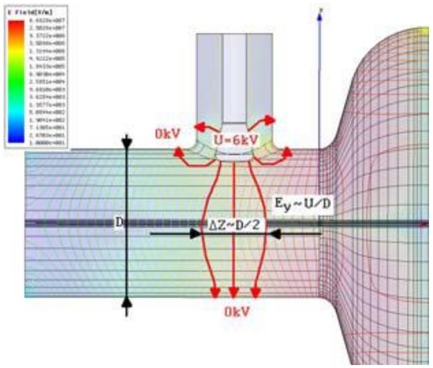
*J. Rosenzweig and L. Serafini, “Transverse Particle Motion in Radio-Frequency Linear Accelerators,” *Phys. Rev. E*, vol. 49, Number 2 (1994).

Focusing properties of RF field

RF acceleration elements (cavities, acceleration structures) typically have no perfect axial symmetry because of design features, coupling elements or manufacturing errors.

- Elliptical SRF cavities have the input couplers, which introduce dipole field components.
- Low-beta cavities like Half-Wave Resonators or Spoke Resonators have quadrupole RF field perturbations cause by spokes, which influence the beam and should be compensated by correctors.





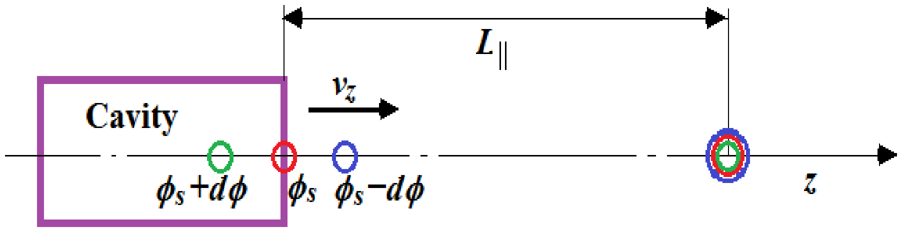
The angle x'_f at the cavity output for x'_i =0 caused by multipole perturbation of m^{th} order (*m*>0) is linear with respect to the ratio of the gain over the particle energy (ultra-relativistic case):

$$x'_f = \frac{\Delta p_\perp}{p_{||}} \approx \frac{m}{ka}\left(\frac{V_{max}(a)}{\gamma m_0 c^2}\right)\left(\frac{x_i}{a}\right)^{m-1}$$

- It may strongly influence the beam dynamics leading to the beam emittance dilution or result in strong quadrupole beam defocusing .
- On the other hand, the octupole perturbations may be used for the cavity alignment.

Bunching of charged particles in electromagnetic field

Because the particle velocity depends on the energy, the cavity RF field provides the beam bunching:



Longitudinal "focusing" distance $L_{//}$:

$$L_{||} = -\frac{2\beta^3\gamma^3}{\omega/c}\frac{m_o c^2/e}{V_{max}(0)\cdot sin\phi_s} = -\frac{1}{2}L_{\perp}$$

For the bunch longitudinal stability $L_{//}$ should be >0, or ϕ_s <0 . In this case, one has transverse defocusing.

Note that for small energy (and therefore small β) the bunching may be too strong, and low RF frequency is to be used for acceleration.

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Example:

SSR1 cavity (PIP II H$^-$ accelerator):

f=325 MHz; V_{max} = 1 MV; φ_s = -34°;

m_0c^2=E_0=938 MeV; E= 10 MeV $\rightarrow \beta \approx (2E/E_0)^{1/2}$=0.146, $\gamma \approx 1$.

❖ The focusing distance $L_\perp$ is

$$L_\perp = \frac{2\beta^3\gamma^3}{\omega/c}\frac{m_oc^2/e}{V_{max}(0)\cdot sin\phi} = \mathbf{-1.55\ m}$$

❖ Longitudinal "focusing" distance $L_{||}$:

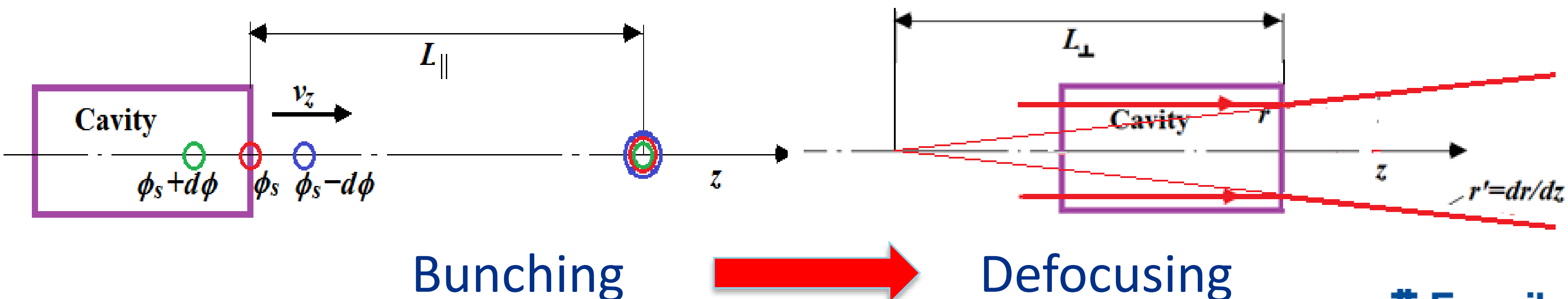
$$L_{||} = -\frac{2\beta^3\gamma^3}{\omega/c}\frac{m_oc^2/e}{V_{max}(0)\cdot sin\phi_s} = -\frac{1}{2}L_\perp = \mathbf{78\ cm}$$

Summary:

- Acceleration of a charged particle moving in axisymmetric RF field parallel to the axis at the radius *r* is proportional to $I_0(kr/\beta\gamma)$;
  - for non-relativistic particle it increases with the radius → for low-energy particles one should use <u>low</u> frequency;
  - for ultra-relativistic particle it does not depend on the radius.
- Focusing of the accelerating particle is related to acceleration;
  - the maximum of transverse momentum change of the non-relativistic particle is shifted in RF phase versus the maximum the energy gain by -90°
  - The focusing distance of the non-relativistic particle is propositional to $\beta^3\gamma^3\lambda/V_{max}$ → for low-energy particles one should use <u>low</u> frequency.

Summary (cont):

- The focusing distance for ultra-relativistic particles is quadratic versus the ratio of particle energy over the voltage.
- The focusing distance for ultra-relativistic particles in multipole fields is linear versus the ratio of particle energy over the voltage; multipole perturbations may strongly affect the beam dynamics.
- The bunching "focusing" distance of the non-relativistic particle is propositional to $\beta^3\gamma^3\lambda/V_{max} \rightarrow$ **for low-energy particles one should use low frequency.**
- The sign is opposite to the focusing: **if the bunch is bunched, it is defocused**, and *vice versa*.
- **In low-energy accelerators external focusing is necessary!**



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Chapter 3.

RF Cavities for Accelerators.

a. Resonance modes – operation mode, High-Order Modes;
b. Pillbox cavity
c. Cavity parameters:
- **Acceleration gradient;**
- **R/Q;**
- **Q_0 and G-factor;**
- **Shunt Impedance;**
- **Field enhancement factors (electric and magnetic);**

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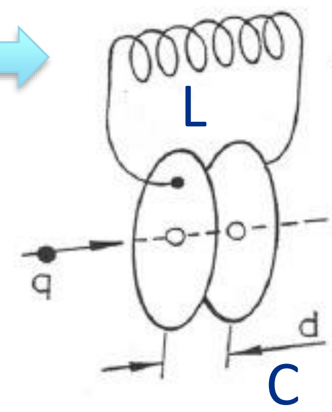
RF cavity:

$\omega_0 = (LC)^{-1/2}$

- ❑ An LC circuit, the simplest form of RF resonator:

This circuit and a resonant cavity share common aspects:

- Energy is stored in the electric and magnetic fields
- Energy is periodically exchanged between electric and magnetic field
- Without any external input, the stored power will turn into heat.



- ❑ To use such a circuit for particle acceleration, it must have opening for beam passage in the area of high electric field (capacitor).
- ❑ As particles are accelerated in vacuum, the structure must provide vacuum space. A ceramic vacuum break (between the two electrode of the capacitor) can be used to separate the beam line vacuum from the rest of the resonator. Or the resonant structure can be enclosed in a vacuum vessel.

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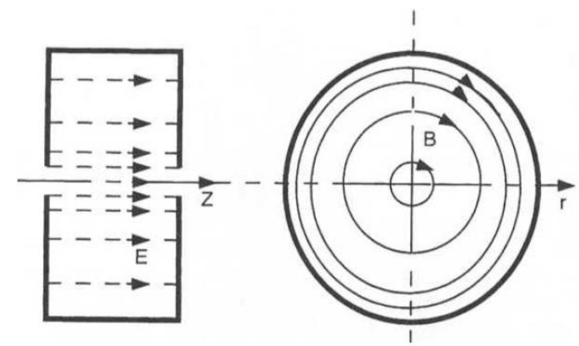
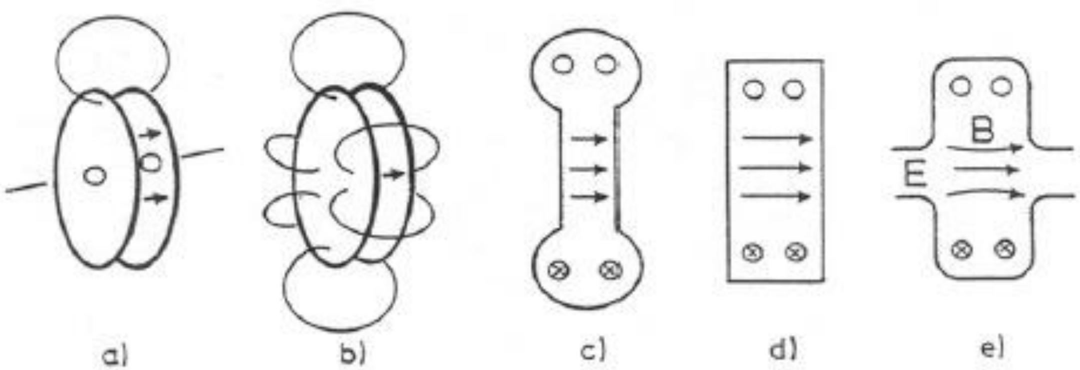
From LC circuit to an accelerating cavity:

- Alternatively, we can use "cavity resonators".
- Metamorphosis of the ***LC*** circuit into an accelerating cavity:

1. Increase resonant frequency by lowering ***L***, eventually have a solid wall.
2. Further frequency increase by lowering ***C*** → arriving at cylindrical, or "pillbox" cavity geometry, which can be solved analytically.
3. Add beam tubes to let particle pass through.





Pillbox geometry:

- Electric field used for acceleration is concentrated near the axis
- Magnetic field is concentrated near the cavity outer wall

Cavity resonators:

A cavity resonator is a closed metal structure that confines electromagnetic fields in the RF or microwave region of the spectrum.

- Such cavities act as resonant circuits with extremely low losses. The Q factor for cavities made of copper is typically of the order of ten thousands compared to a few hundreds for resonant circuits made with inductors and capacitors at the same frequency.

- Resonant cavities can be made from closed (or short-circuited) sections of a waveguide or coaxial line. Ferrite-loaded cavities are used at low frequencies to make cavities compact and allow very wide frequency tuning range.

- The cavity wall structure can be made stiff to allow its evacuation.

- Electromagnetic energy is stored in the cavity and the only losses are due to finite conductivity of cavity walls and dielectric/ferromagnetic losses of material filling the cavity.

Modes in an RF cavity:

$$\Delta\vec{E} + k^2\vec{E} = 0, \quad \Delta\vec{H} + k^2\vec{H} = 0.$$

where $k = \omega\sqrt{\mu\varepsilon}$

Boundary conditions

$$\vec{n} \times \vec{E} = 0$$

$$\vec{n} \cdot \vec{H} = 0$$

- There are an infinite number of orthogonal solutions (eigen modes) with different field structure and resonant frequencies (eigen frequencies).
- For acceleration in longitudinal direction the lowest frequency mode having longitudinal electric field component is used.

Properties of resonance modes:

- Relation between eigenvalue k_m and eigenfunction $\vec{H}_m$:

$$k_m^2 = \frac{\int_V |\mathrm{rot}\, \vec{H}_m|^2 \, dV}{\int_V |\vec{H}_m|^2 \, dV}. \qquad \omega_m = ck_m = \frac{k_m}{\sqrt{\mu\varepsilon}}, \quad \lambda_m = \frac{2\pi}{k_m}.$$

- The eigen functions are orthogonal

$$\int_V \vec{E}_m \cdot \vec{E}_n \, dV = 0, \quad \int_V \vec{H}_m \cdot \vec{H}_n \, dV = 0 \qquad \text{if} \quad k_m^2 \neq k_n^2$$

- The <u>average</u> energies stored in electric and magnetic fields are equal:

$$\int_V \frac{\mu|H_m|^2}{4} dV = \int_V \frac{\varepsilon|E_m|^2}{4} dV = \frac{W_0}{2} \Rightarrow W_0 = \int_V \frac{\mu|H_m|^2}{4} dV + \int_V \frac{\varepsilon|E_m|^2}{4} dV = \int_V \frac{\mu|H_m|^2}{2} dV = \int_V \frac{\varepsilon|E_m|^2}{2} dV$$

- The eigenmode variation property: for small cavity deformation one has:

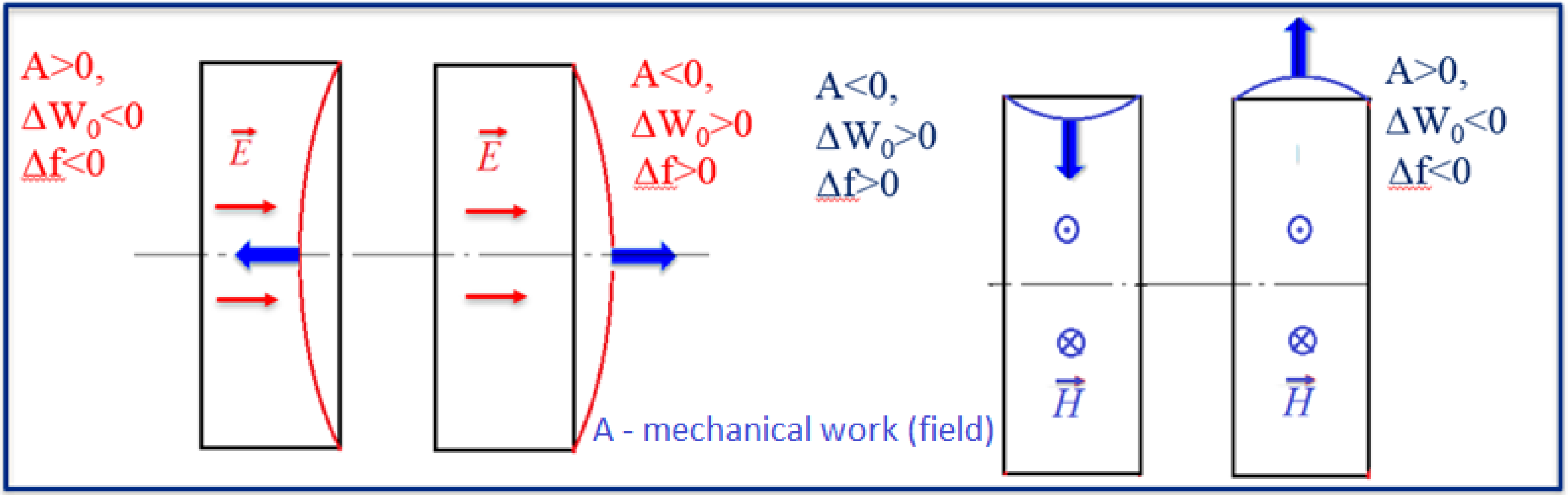
$$\frac{W_0}{\omega_0} = \mathrm{const}$$

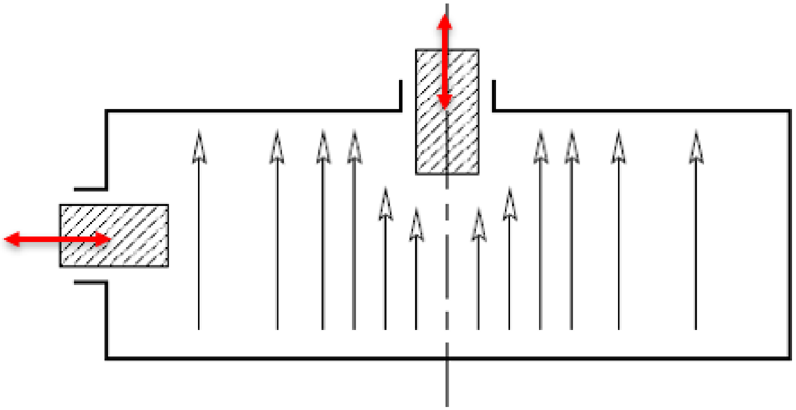
W_0 us stored energy, ω_0 is the mode circular resonance frequency.

See for details <u>Appendix 3</u>

Properties of resonance modes :

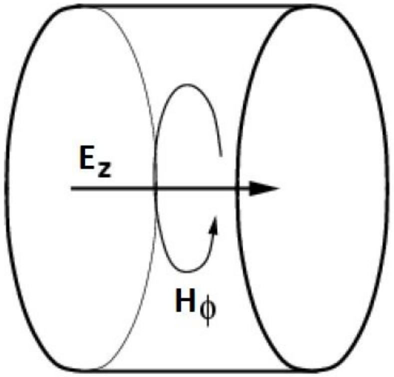
Cavity mechanical tuning is based on the eigenmode variation property:





Resonance modes and pillbox cavity:

- Most of acceleration cavities have axial symmetry (slightly violated by perturbations – coupling units, manufacturing errors, etc).
- The modes in the axisymmetric cavity of arbitrary shape have azimuthal variations, $\vec{E}, \vec{H} \sim exp(im\phi)$:
  - -For acceleration TM-modes with m=0 ("monopole") are used;
  - -Dipole (m=1) TM-modes are used for the beam deflection.
- The simplest cavity is a pillbox one:



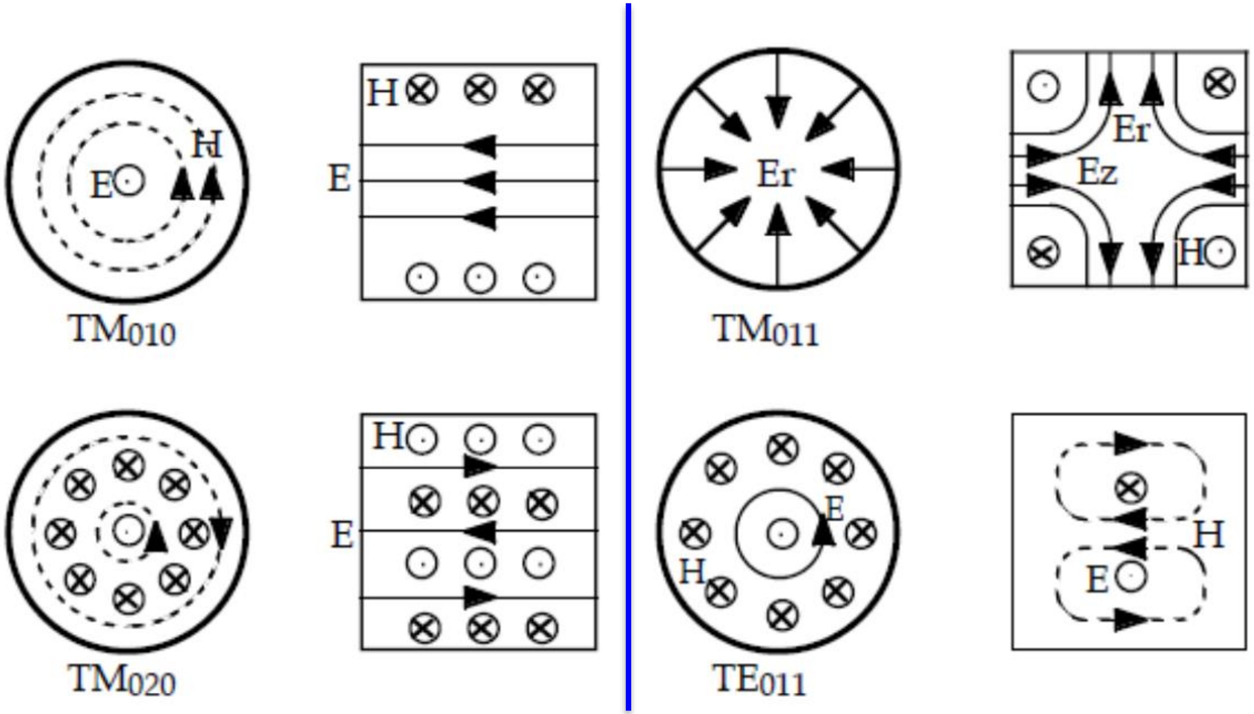
Resonance modes and pillbox cavity:

- For pillbox cavities there are two families of the eigen modes:
  - TM -modes, which have no longitudinal magnetic fields;
  - TE-modes, which have no longitudinal electric fields.
- The modes are classified as TM_{mnp} (TE_{mnp}), where integer indices *m, n,* and *p* correspond to the number of variations E_z (H_z) has in φ, r, and z directions respectively (see Appendix 5).

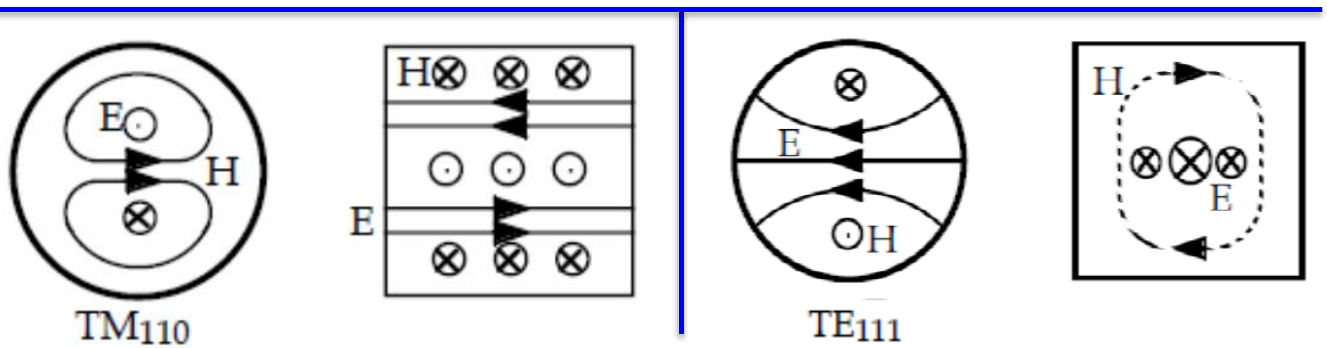
- For "monopole" modes in the axisymmetric cavity of arbitrary shape
  - TM-modes have only azimuthal magnetic field component;
  - TE -modes have only azimuthal electric field component.

  For acceleration, lowest TM-mode is used, which has longitudinal electric filed on the axis.

Resonance modes and pillbox cavity:



Modes with m=0 ("monopole modes")



Modes with m=1 (dipole modes)

Modes in a pillbox RF cavity:

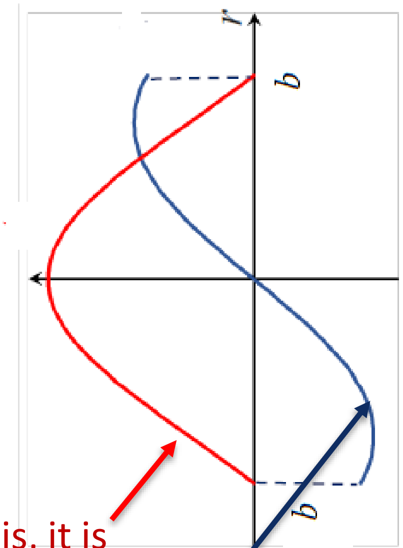
- While TM_{010} mode is used for acceleration and usually is the lowest frequency mode, all other modes are "parasitic" as they may cause various unwanted effects. Those modes are referred to as High-Order Modes (HOMs). Modes with m=0 – "monopole", with m=1 – "dipole", etc.

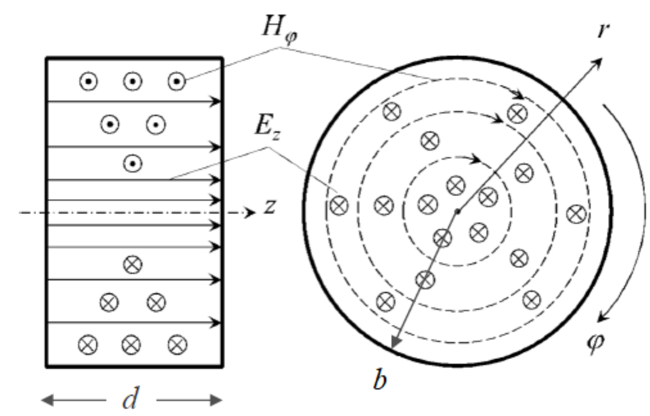
$$E_z = E_0 J_0\left(\frac{2.405r}{b}\right)e^{i\omega t}$$

$$H_\phi = -i\frac{E_0}{Z_0}J_1\left(\frac{2.405r}{b}\right)e^{i\omega t}$$

$$\omega_{010} = \frac{2.405c}{b},\quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\lambda_{010} = 2.61b$$



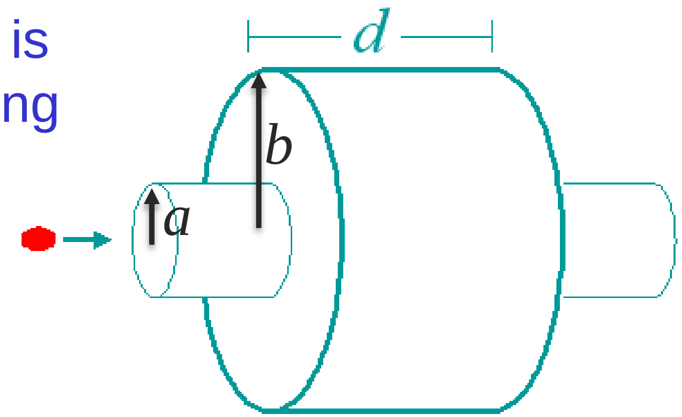


- Electric field is concentrated near the axis, it is responsible for acceleration.
- Magnetic field is concentrated near the cylindrical wall, it is responsible for RF losses.

Note that electric and magnetic fields are shifted in phase by 90 deg. For vacuum Z_0=120π Ohms; b is the pillbox radius, d is its length.

Accelerating voltage and transit time factor*:

Assuming charged particles moving along the cavity axis, and the particle velocity change is small, one can calculate *maximal* accelerating voltage V as



$$V = \left| \int_{-\infty}^{\infty} E_z(r=0,z) e^{i\omega_0 z/\beta c} dz \right|$$

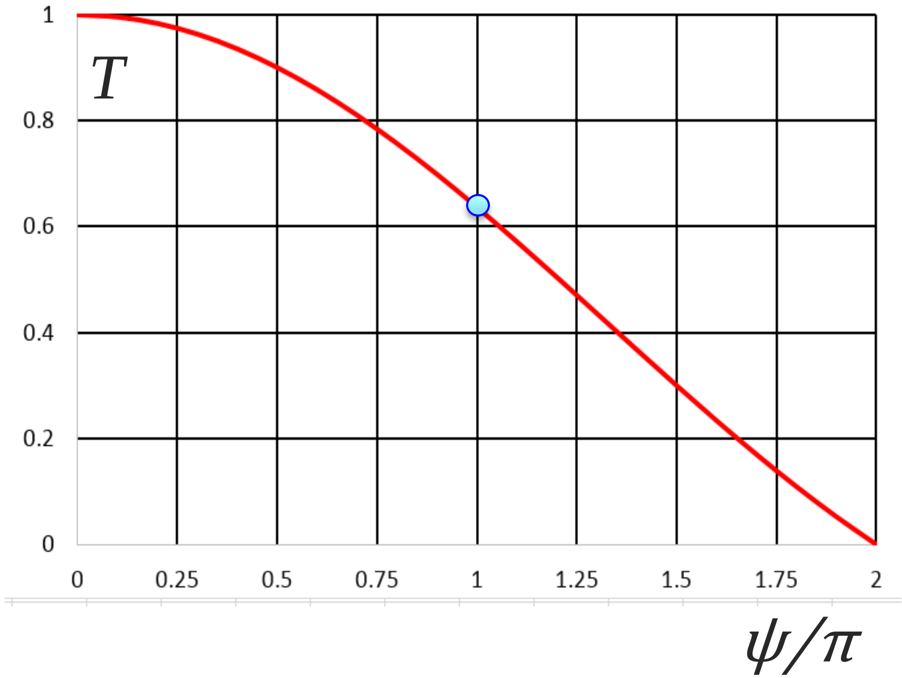
For the pillbox cavity one can integrate this analytically:

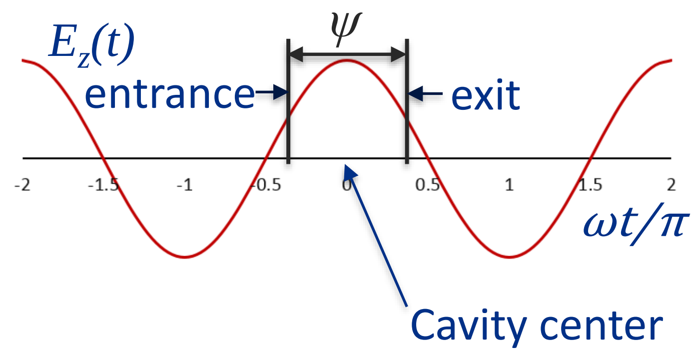
$$V = E_0 \left| \int_0^d e^{i\omega_0 z/\beta c} dz \right| = E_0 d \frac{\sin\left(\frac{\omega_0 d}{2\beta c}\right)}{\frac{\omega_0 d}{2\beta c}} = E_0 d \cdot T$$

where T is the transit time factor, $T(\psi) = \frac{\sin(\psi/2)}{\psi/2}, \ \psi = \frac{\omega_0 d}{\beta c}$

***Details are in Appendix 4**

Acceleration gradient





Note that maximal acceleration takes place when the RF field reaches maximum when the particle is the cavity center.

In order to "use" all the field for acceleration , $\psi=\pi$ (or $d = \beta\lambda/2$) and $T = 2/\pi$ for the pill box cavity. $\lambda = 2\pi c/\omega_0$– wavelength.

- ***Acceleration gradient*** $\mathbf{E}$ ***is defined as*** $\mathbf{E=V/d=E_0T}$

Unfortunately, the cavity length is not easy to specify for shapes other than pillbox so usually it is assumed to be ***d = βλ/2****. This works OK for multi-cell cavities, but poorly for single-cell cavities or cavities for slow particle acceleration.*

RF cavity parameters:

- Stored energy U:

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv = \frac{1}{2}\varepsilon_0 \int_V |\mathbf{E}|^2 dv$$

- Losses in the cavity. There are the losses P_c in a cavity caused by finite surface resistance R_s:

$$P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 ds$$

For normal conducting metal at room temperature (no anomalous skin effect)

$R_s = \dfrac{1}{\sigma\delta}$, where σ is conductivity and δ is skin depth, $\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}}$

Example:

For copper at room temperature σ=59.6 MS/m; R_s= 9.3 mOhm@1.3 GHz

Simple formula for estimation: $\delta = 0.38 \cdot (30/f(\mathrm{GHz}))^{1/2}$ [mkm]

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RF cavity parameters:

Unloaded quality factor Q_0:

$$Q_0 \equiv \frac{\omega_0 \cdot (\text{stored energy})}{\text{average power loss}} = \frac{\omega_0 U}{P_c} = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}$$

Quality factor Q_0 roughly equals to the number or RF cycles times 2π necessary for the stored energy dissipation.

One can see that

$$Q_0 = \frac{G}{R_s}$$

where G is so-called geometrical factor (same for geometrically similar cavities),

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

RF cavity parameters:

For a pillbox cavity:

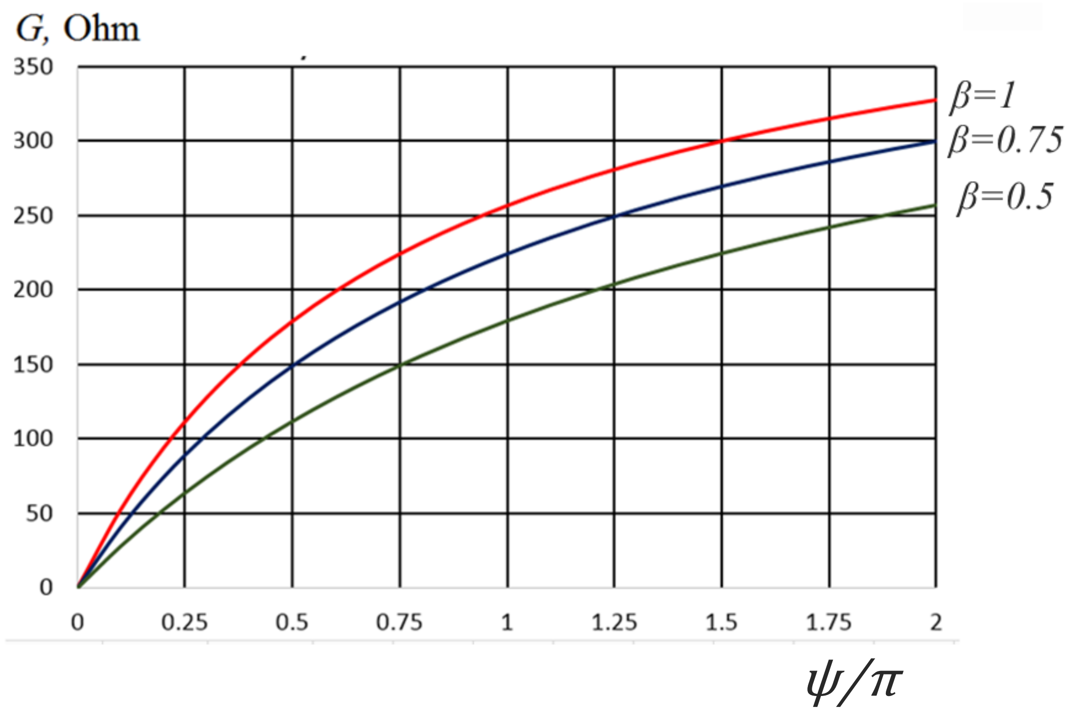
$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

$$H_\varphi = J_1(kr), \; k = \frac{\omega_{010}}{c} = \frac{2.405}{b}$$

$$\int_V |\vec{H}_m|^2 \, dV = \pi d b^2 J_1^2(kb), \quad \oint_S |\vec{H}_m|^2 \, dS = 2\pi b(b + d) J_1^2(kb)$$

and

$$G = 1.2 Z_0 \frac{1}{1 + b/d}.$$



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RF cavity parameters:

For a room-temperature pillbox cavity

$$Q_0 = \frac{1}{\delta}\frac{bd}{b+d}.$$

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}$$

- For pillbox having $\psi=\pi$ and $\beta=1$ $(d = \lambda/2 = \pi b/2.405)$, G =257 Ohms.
- Therefore, 1.3 GHz RT copper pillbox cavity has Q_0 = 2.6e4.
- For 1.3 GHz SRF Nb cavity at 2K one has Q_0 = 3e10 (R_s= 8.5 nOhm).

For geometrically similar RT cavities Q_0 scales as $f^{-1/2}$ *or* $\lambda^{1/2}$!

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RF cavity parameters:

Estimation of the unloaded Q_0 for an arbitrary room-temperature cavity:

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\vec{H}|^2 \, dV}{R_s \int_S |\vec{H}|^2 \, dS}, \quad R_s = \frac{1}{\sigma\delta},$$

Taking into account that $\omega_0 \sigma = \frac{2}{\delta^2 \mu_0}$ one has:

$$Q_0 = \frac{2}{\delta} \frac{\int\limits_V |\vec{H}|^2 \, dV}{\oint\limits_S |\vec{H}|^2 \, dS}.$$

One may introduce the average surface and volume fields:

$$Q_0 = \frac{2}{\delta} \frac{V\,|H\,|_V^2}{S\,|H\,|_S^2}, \quad 2\,|H\,|_V^2 = A\,|H\,|_S^2.$$

for accelerating mode $A \sim 1$

For convex figures $V/S \sim a_{av}/6$ (cube: $V/S = a/6$. sphere: $V/S = 2R/6$) and

$$Q_0 \approx \frac{1}{6} \frac{a_{av}}{\delta} A.$$

Note, that $a_{av} \sim \lambda$, $\delta \sim \sqrt{\lambda}$ and $Q_0 \sim \lambda^{1/2}$,

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RF cavity parameters:

- An important parameter is the cavity shunt impedance R, which determines relation between the cavity accelerating voltage V and power dissipation:

$$R = \frac{V^2}{P_c}$$

- Another important parameter is (R/Q), which determines relation between the cavity voltage V and stored energy U. Itis necessary to estimate the mode excitation by the accelerated beam. It does not depend on the surface resistance and is the same for geometrically similar cavities:

$$\frac{R}{Q} = \frac{V^2}{\omega_0 U} = 2\frac{\left|\int_{-\infty}^{\infty} E_z(\rho = 0, z) e^{i\omega_0 z/\beta c} dz\right|^2}{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}$$

Note that $R = \frac{R}{Q} \cdot Q_0$ and power dissipation $P_{diss} = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = \frac{V^2 \cdot R_s}{\frac{R}{Q} \cdot G}$

*Sometimes they use a "circuit definition: $\frac{R}{Q} = \frac{V^2}{2\omega_0 U}$

RF cavity parameters:

For a pillbox cavity:

$$V = E_0 d \cdot T(\psi),$$

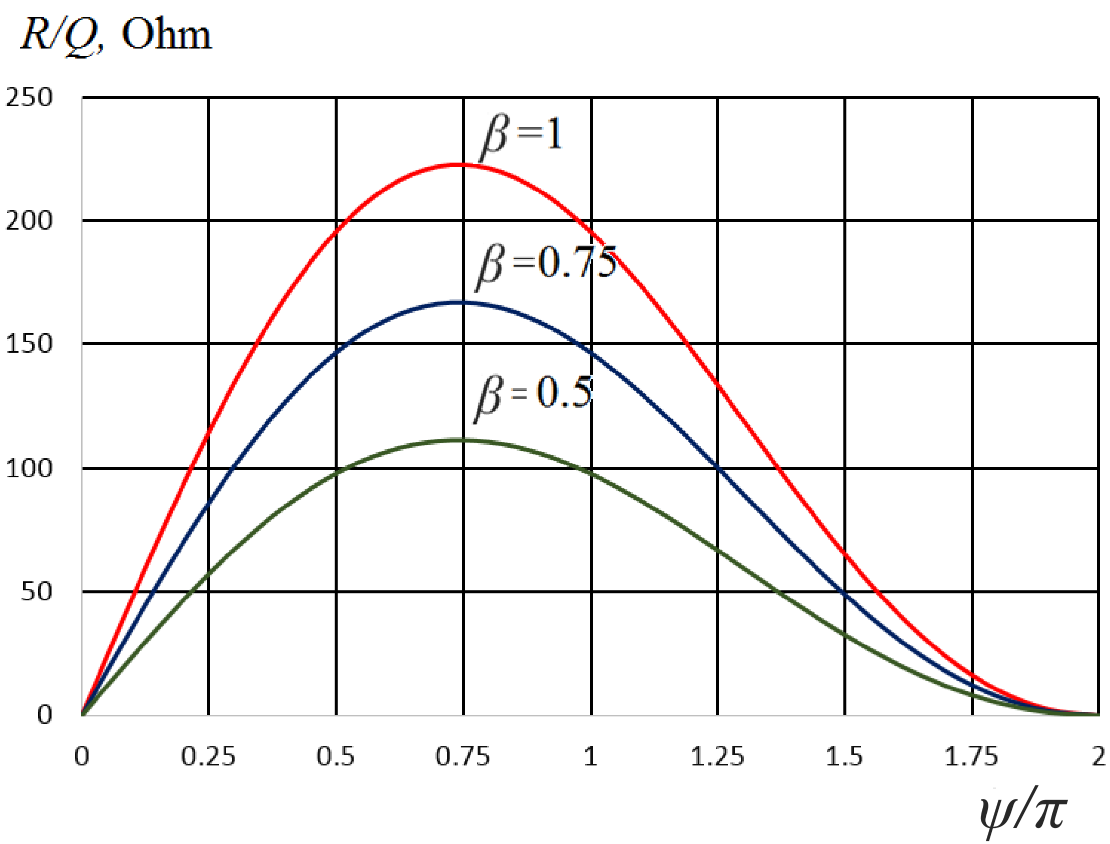
$$\int_V |\vec{H}_m|^2 \, dV = \frac{E_0^2}{Z_0^2} \pi d b^2 J_1^2(kb),$$

$$J_1(kb) = J_1(2.405) = -0.519$$

$$\omega_0 \mu_0 = Z_0 2.405/b$$

and

$$\frac{R}{Q} = 0.98 Z_0 d/b \cdot T^2(\psi)$$



- For pillbox having $\psi=\pi$ and $\beta=1$ ($d=\lambda/2=\pi b/2.405$), R/Q =196 Ohms.
- R/Q is maximal for for $\psi\approx 2\pi/3$.

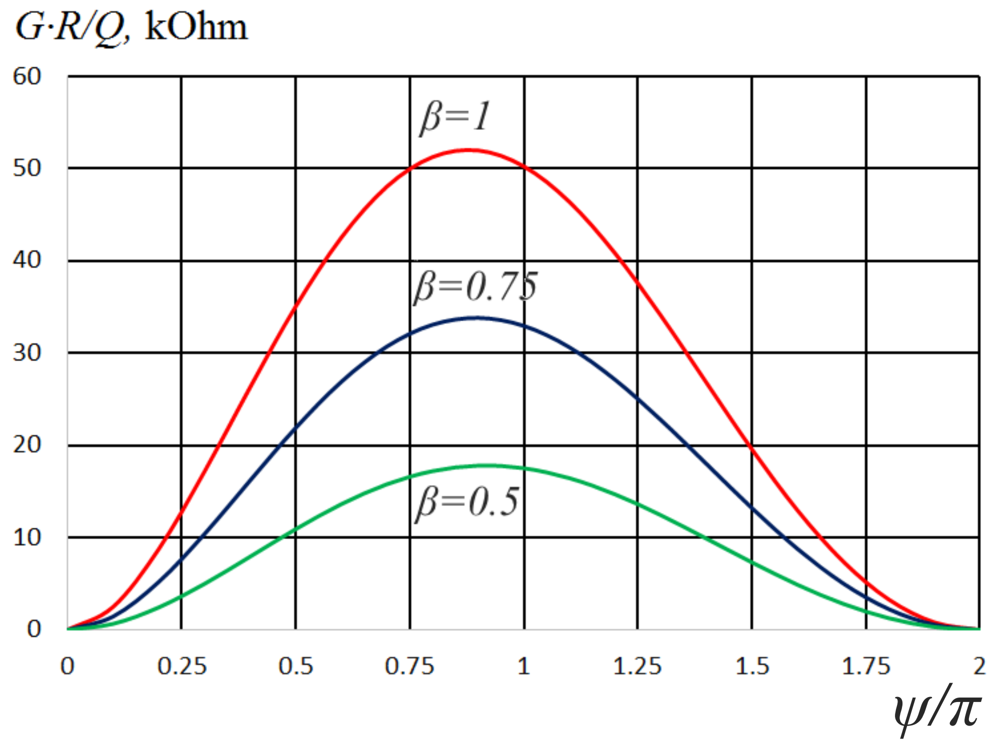
RF cavity parameters:

The power loss in the cavity walls is

$$P_c = \frac{V^2 \cdot R_s}{G \cdot (R/Q)}$$

Therefore, the losses are determined by $G \cdot R/Q$.

For pillbox:



$G \cdot R/Q$ is maximal for $\psi \approx 0.9\pi$

RF cavity parameters:

Gradient limitations are determined by surface fields:

- RT cavities:
  - -breakdown (determined mainly by E_{peak})
  - -metal fatigue caused by pulsed heating (determined by B_{peak})
- SRF cavities:
  - -quench (determined by B_{peak})

Field enhancement factors:

- Surface electric field enhancement: $K_e = E_{peak}/E$, E_{peak} is maximal surface electric field. K_e *is* dimensionless parameter.

- Surface magnetic field enhancement: $K_m = B_{peak}/E$, B_{peak} is maximal surface magnetic field. K_m is in mT/(MeV/m)

Here E is acceleration gradient.

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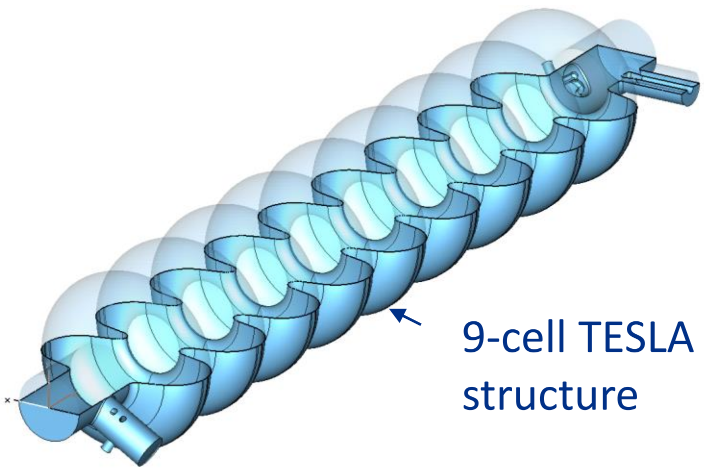
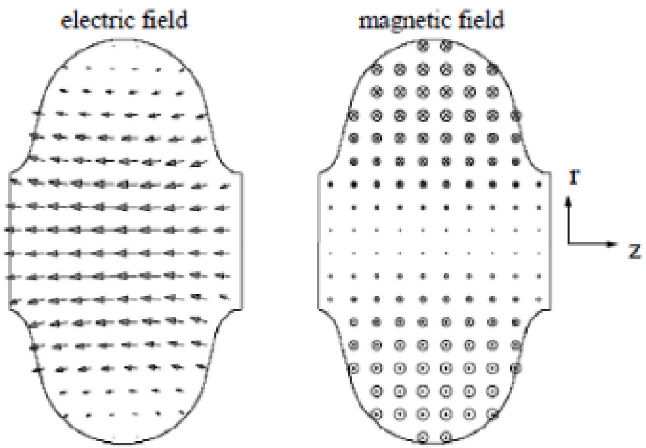
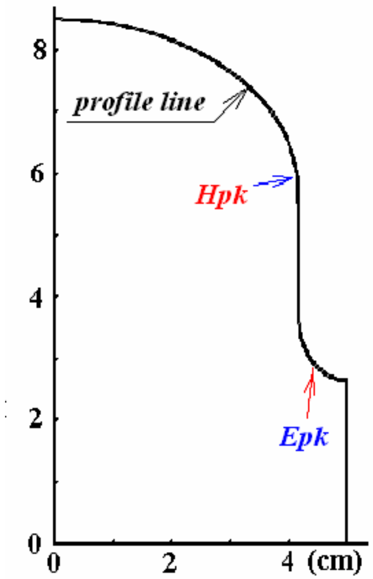
RF cavity parameters:

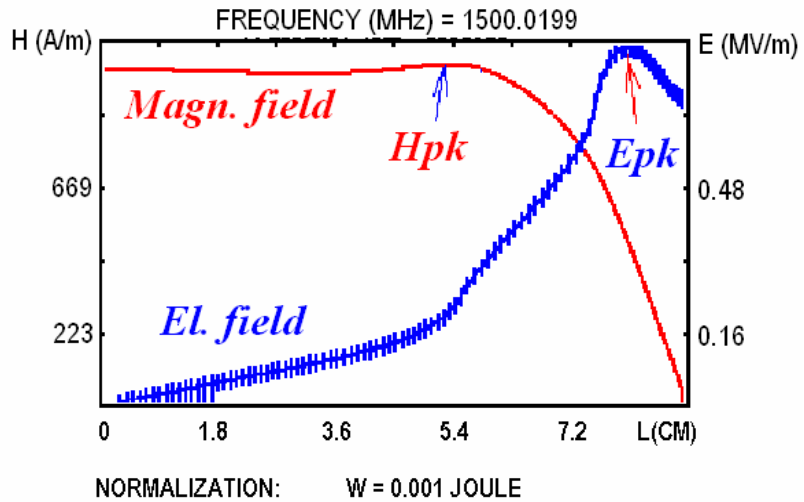
Field enhancement factors – example:



9-cell TESLA structure







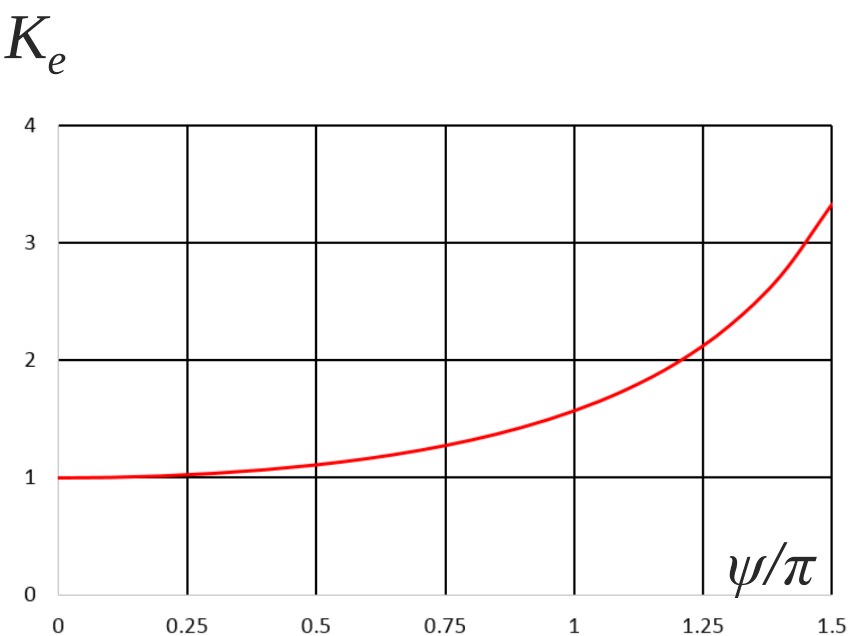
$$K_e = E_{peak}/E = 2; \quad K_m = B_{peak}/E = 4.16\ mT/MV/m$$

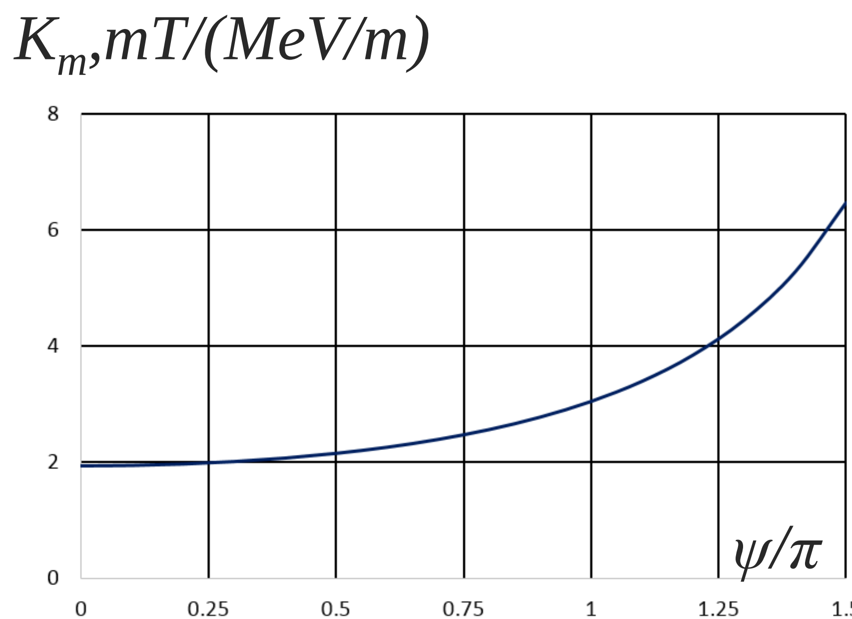
Geometry of an inner half-cell of a multi-cell cavity and field distribution along the profile line.

RF cavity parameters:

For a pillbox cavity:

- Surface electric field enhancement: $K_e = E_{peak}/E = 1/T(\psi)$
  ($E_{peak}=E_0$, $E = E_0 T(\psi)$, see slide 29)
- Surface magnetic field enhancement: $K_m = B_{peak}/E =$
  $= 1.94/T(\psi)$ [mT/(MeV/m)]
  ($B_{peak} = E_0 \cdot J_1(2.405r/b)_{max}/c = 0.582E_0/c$, see slide 46)





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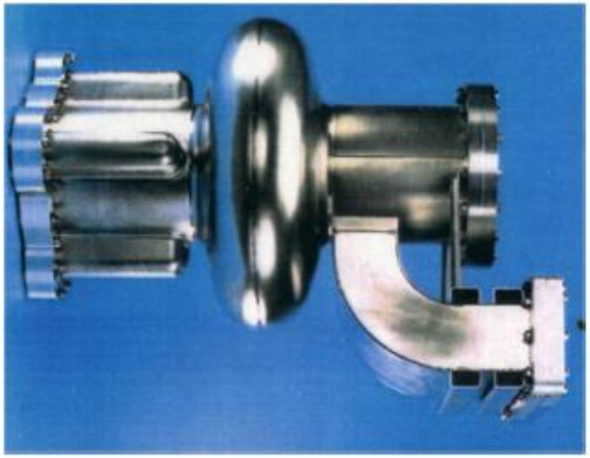
Pillbox *vs.* “real life” SC cavity

Quantity	Cornell SC 500 MHz	Pillbox
G	270 Ω	257 Ω
R/Q	88 Ω/cell	196 Ω/cell
$E_{\rm pk}/E_{\rm acc}$	2.5	1.6
$B_{\rm pk}/E_{\rm acc}$	$5.2\,mT/(\mathrm{MV/m})$	$3.05\,mT/(\mathrm{MV/m})$



Cornell SC 500 MHz

- In “real life” cavities, sometimes it is necessary to damp higher-order modes (HOMs) to avoid beam instabilities.
- The beam pipes are made large to allow HOMs propagation toward microwave absorbers.
- This enhances B_{pk} and E_{pk} and reduces R/Q.

Example:

Let's consider a pillbox cavity for high-energy electrons ($\beta\approx 1$), f=500 MHz, or wavelength $\lambda=c/f$=0.6 m. The mode is TM_{010}. The cavity voltage V is 3 MV.

1. The cavity radius b (Slide 49):

   $b= 2.405c/(2\pi f) = 230\ mm.$

2. The cavity transit time factor for $\psi=\pi$ (Slide 50):

   $T=\sin(\pi/2)/(\pi/2) = 2/\pi =0.64.$

3. The cavity length d (Slide 51):

   $d = \beta\lambda/2 =300\ mm.$

4. The cavity G- factor (Slide 54):

   $G=1.2Z_0/(1+b/d) = 256\ Ohm$

5. The cavity R/Q (Slide 58):

   $R/Q= 0.98Z_0(d/b)T^2=196\ Ohm$

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Example:

6. Surface resistance R_s for room-temperature copper (Slide19):

$$R_s = \sqrt{\frac{\omega Z_0}{2c\sigma}} = 5.8\ mOhm$$

7. Surface resistance R_s for superconducting Nb at 2 K (Slide 23):

$$R_{s,BCS} \approx 1.643 \times 10^{-5} \frac{T_c}{T} (f(GHz))^2 e^{-\frac{1.92 T_c}{T}} = 2.8\ nOhm$$

8. Copper cavity unloaded quality factor Q_0 (Slide 53):

$Q_0 = G/R_s = 44e3$

9. Nb cavity unloaded quality factor Q_0 at 2 K (Slide 53):

$Q_0 = G/R_s = 9e10$ *(compared to 44e3 for copper cavity!)*

Example:

10. Copper cavity wall power dissipation (Slide 57):

$$P_{diss} = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = 1\ MW$$ – *unacceptable!*

11. Nb cavity wall power dissipation (Slide 57):

$$P_{diss} = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = 0.5\ W!$$

12. Acceleration gradient (Slide 51):

$$E = V/d = 3\ MeV/0.3m = 9\ MeV/m$$

13. Peak surface electric and magnetic fields (Slide 62):

$$E_{peak} = K_e \cdot E = E/T = 14.1\ MV/m = 141\ kV/cm$$ – *OK for SC*

$$B_{peak} = K_m \cdot E = 1.94 \cdot E/T\ mT = 27\ mT$$ – *OK for SC*

SC cavities allow much higher acceleration gradient at CW!

Summary:

- To create acceleration RF field, resonance RF cavities are used;
- The cavities typically have axisymmetric field distribution near the beam axis. Most of cavities have geometry close to axisymmetric.
- There are infinite number of resonance modes in an RF cavity having different radial, azimuthal and longitudinal variations. The modes are orthogonal;
- In axisymmetric cavities there are two types of modes, TM and TE;
- For acceleration, TM_{010} mode is used, which has axial electric field on the axis.
- Other modes, HOMs, are parasitic, which may caused undesirable effects.

Summary (cont):

- The cavity mode is characterized by the following parameters:

- Resonance frequency;
- Acceleration gradient (energy gain/cavity length);
- Unloaded Q, Q_0 , which characterize the losses in the cavity;
- *G*-factor, which relates Q_0 and surface resistance;
- *(R/Q)*, which relates the energy gain and the energy stored in the cavity;
- Shunt impedance R, which relates the gain and total losses in the cavity;
- Electric and magnetic field enhanced factors, which relate maximal surface fields and the acceleration gradient;